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SHORT COURSE IN Arithmetic

CLIFFORD B. UPTON

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SHORT COURSE IN ARITHMETIC

Principles of Arithmetic, Graded Practice, Diagnostic Tests; and Remedial Exercises

By CLIFFORD B. UPTON

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INFORMATION FOR TEACHERS

Scope. This book covers the fundamental operations of arithmetic, with clear explanations of all difficult topics. It harmonizes with the recommendations on arithmetic of recent Yearbooks of the National Council of Teachers of Mathematics.

Purposes Served. This book may be used in any of the following ways:

(1) To provide a review course in arithmetic in any year of the high school, including the junior high school.

(2) To supply supplementary material for use in connection with any other class in high school mathematics.

(3) To provide a rapid review of the subject matter of arithmetic in normal schools and teachers colleges preparatory to, or in conjunction with, courses on methods of teaching arithmetic.

(4) To provide a short course in arithmetic for adults in evening and extension classes.

Flexibility. The book is suitable for use with an entire class where the students are kept together on each topic, or for individual use where each student progresses at his own pace. It is also adapted for self-instruction.

Improvement Tests. Sixty Improvement Tests are distributed throughout this book. By means of these tests it is possible, in an interesting way, to maintain and to perfect the computational skills with whole numbers that the students have already learned. These tests take less than a half hour each week, thus leaving ample time for the study of other topics. For instructions for using these tests, see pages 138–140.

Scoring. By means of the Scoring Table, given on the inside of the back cover of this book, the student can quickly score any Improvement Test on a scale of 10. These scores are recorded on a graphic chart on page 166, thus making it possible for the student easily to determine whether or not he is gaining in computational skill.

Diagnostic Tests. Diagnostic Tests are distributed throughout the book, which enable the teacher to discover those types of work upon which the student needs further practice. These Diagnostic Tests have keyed references to the remedial exercises which are described in the next paragraph.

Remedial Work. A comprehensive series of remedial exercises, covering whole numbers, fractions, decimals, and percentage, is given on pages 145–163 of this book; these exercises provide additional practice for those students that need it. References to this remedial work are given on the Diagnostic Tests and at the bottom of many of the other pages of this book. See page 138.

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11 13 EP 12 10

ADDITION

Add upward. Check the work by adding downward:

	a	*b*	*c*	*d*	*e*
(1)	6825 7132 13957	4231 2768 6999	36554 14372 50926	62284 78331 140615	70010 24791 94801

Test on Addition. Add upward. Check by adding downward: Time 4 min.

(2)	7491 5951 13442	4319 9168 13487	25097 15470 40567	38059 25659 63718	67270 57380 124650
(3)	7004 4598 13602	1805 3489 5294	81623 24515 106138	77949 40122 118071	37655 18057 55712
(4)	1253 3478 4731	6237 1933 8170	93318 56957 150275	36642 77187 113829	40634 25296 65930
(5)	4658 5399	2689 6000	19049 13244	76534 69387	53127 26888

14 RIGHT OUT OF 20 SCORE 7 LINE T

Scoring Exercises. In the test above there are 20 examples. Mark the number right in the space at the left; then insert your score in the space at the right. To find your score, use the Scoring Table on the inside of the back cover. If your score is 10, your work is satisfactory.

How to Improve in Addition. If your score is 9, you will profit by more practice, which will be found on Remedial Sheet 6 on page 150. If your score is below 9, it is probable that you have not thoroughly learned all the 100 fundamental addition facts which are given on Remedial Sheet 1 on page 145. Practice on these addition facts until you have mastered them. It will save time if you make a list of the particular facts that cause you trouble and confine your practice to those facts. After such practice, take again the test on this page and see if you can improve your score. Also do the work on Remedial Sheet 6 on page 150.

NOTE. See directions for finding scores on the inside of the back cover.

To add a column of figures like the one at the right, begin at the bottom of the column and add up as follows: $7 + 5 = 12$, $12 + 4 = 16$, $16 + 7 = 23$, $23 + 8 = 31$. In actual practice, give the *results only*, like this: "12, 16, 23, 31." Check the work by adding down, thus: "15, 19, 24, 31."

8	
7	
4	
5	
7	
31	Sum

To add a column rapidly you have to be very skillful in the *mental addition* of a one-figure number to a two-figure number. For example, in adding the column at the right, you had to add 12 and 4, 16 and 7, 23 and 8, and all this work had to be done mentally; such mental addition is called *adding by endings*.

In addition, the numbers that are added are called *addends* and the answer is called the *sum*.

How to Improve Your Score. If your score on the addition test below is 8 or less, you probably need considerable practice on adding by endings which is given on Remedial Sheet 2 on page 146; you will also profit by practice on the addition facts given on Remedial Sheet 1 on page 145.

Add upward. Check the work by adding downward:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
(1)	5	3	1	8	9	38	86	93	63
	3	7	7	2	6	21	75	31	50
	6	8	1	0	6	59	14	47	15
	2	1	9	3	8	42	34	81	34
	8	3	9	0	8	66	77	99	18
	9	3	1	6	0	18	53	24	49
	2	7	5	9	3	20	96	15	60

Test on Addition. Add upward. Check by adding downward: Time 3 min.

(2)	7	9	6	3	31	42	69	35	31
	6	8	7	9	29	67	25	20	58
	9	8	0	8	86	63	52	41	11
	2	1	5	0	44	38	26	16	90
	8	4	6	2	52	29	75	14	35
	7	1	0	4	70	95	69	68	99
	4	6	8	9	63	65	58	73	74

RIGHT OUT OF 9

SCORE ..... LINE H

NOTE. See page 138 for suggestions on the use of the Remedial Sheets.

COLUMN ADDITION

Add upward. Check the work by adding downward:

(1)

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*
5	2	4	8	3	8	4	5	6	7	4	7
3	6	5	2	2	3	6	7	3	4	4	5
2	8	4	1	2	0	1	2	1	6	3	4
8	4	8	6	7	7	7	7	0	4	5	4
8	4	3	3	3	7	2	4	9	8	6	6
1	3	9	7	4	2	2	1	9	0	6	5
2	9	1	4	0	6	9	3	2	9	0	2
5	6	3	4	3	0	0	0	1	4	4	6
1	5	0	1	5	8	4	9	3	7	4	9
5	5	7	6	0	9	6	9	8	1	7	7
40	52	44	42	29	50	41	47	42	50	43	55

(2)

a	*b*	*c*	*d*	*e*	*f*	*g*
41	18	37	22	14	63	24
27	26	21	61	58	10	82
19	49	85	46	89	56	75
55	72	33	57	41	15	50
66	75	39	60	78	80	78
30	86	95	96	62	57	61
62	52	26	46	36	82	53
36	9	12	52	72	12	36
47	13	21	67	30	9	62
19	61	43	19	65	48	40
302	461	412	526	595	432	561

(3)

108	193	612	183	139	841	209
294	708	455	250	527	196	644
512	227	432	344	802	213	443
966	413	321	69	207	307	237
300	106	400	261	374	686	49
115	260	235	983	795	492	339
468	175	868	405	181	181	653
582	776	99	114	575	45	520
695	898	798	759	867	130	484
568	159	256	624	417	391	556
4407	3918	4476	3992	5084	3482	4134

---- RIGHT OUT OF 7

SCORE ---- LINE F

For more practice, see page 151.

SUBTRACTION

Subtract. Check the work by adding up:

	a	*b*	*c*	*d*	*e*
(1)	4829 3615	6255 849	94164 93889	60000 35478	80004 38296

Test on Subtraction. Subtract. Check the work by adding up: Time 4 min.

(2)	9538 1645	4601 3587	81833 44179	72100 4009	66293 27413
(3)	6369 2709	1500 1149	37703 19168	63144 47864	80000 44682
(4)	7000 2563	3639 2642	61452 54875	80021 25906	34518 1858
(5)	9322 6843	2000 643	72346 53487	10000 8726	71147 41276

_____ RIGHT OUT OF 20 SCORE _____ LINE T

How to Improve Your Score in Subtraction. If your score on the above test is below 9, you probably have not thoroughly learned all the 100 fundamental subtraction facts which are given on Remedial Sheet 3 on page 147. Practice these subtraction facts until you have mastered them. It will save time to make a list of the particular facts that cause you trouble and to confine your practice to them. After practicing on these facts, repeat the test above and see if you can improve your score.

427 Minuend
138 Subtrahend
289 Remainder or Difference

In subtraction the number from which you subtract is called the *minuend;* the number subtracted is called the *subtrahend;* and the answer is called the *difference* or *remainder*. The best way to check subtraction is to add the remainder to the subtrahend, adding up. If the result equals the minuend, the work is correct.

For more practice, see page 150.

Improvement Test — Addition A1

Time 4 min.

	a	*b*	*c*	*d*	*e*	*f*
(1)	568	512	705	416	895	434
	384	243	352	395	293	691
	416	611	103	131	440	319
	221	235	761	828	609	562
	474	300	247	772	227	618
	606	474	813	359	850	144
	743	756	495	987	286	590
	152	628	365	200	129	327

-------- RIGHT OUT OF 6 SCORE -------- LINE E

NOTE. For directions for taking and scoring Improvement Tests, see pages 138–140.

Subtraction. Subtract. Check the work by adding upward:

	a	*b*	*c*	*d*	*e*
(2)	9328	60010	71364	822266	200000
	1189	44785	34897	185349	189954
(3)	9774	77426	96331	409238	812202
	479	73928	23289	191458	339536
(4)	3000	25000	94569	530000	732143
	2816	10073	75874	269082	496846
(5)	7466	11839	50818	616916	476000
	4769	2164	33399	378966	190004
(6)	2316	50000	63745	738074	464857
	1438	29965	8822	300095	285848

For more practice, see pages 150 and 151.

MULTIPLICATION

Multiply. Check the work by going over it again:

	a	b	c	d	e	f
(1)	2978 × 5 = 14890	9657 × 2 = 19314	2438 × 7 = 17066	4253 × 4 = 17012	1368 × 6 = 8208	3569 × 9 = 32121
(2)	3689 × 3 = 11067	2078 × 9 = 18702	4615 × 5 = 23075	3964 × 4 = 15856	4698 × 8 = 37584	7593 × 3 = 22779

Test on Multiplication. Multiply the following: Time 4 min.

(3)	2189 × 8 = 17512	1892 × 6 = 11352	3342 × 5 = 16710	5824 × 2 = 11648	3942 × 7 = 27594	7816 × 4 = 31264
(4)	1756 × 7 = 12292	8596 × 2 = 17192	6514 × 9 = 58626	2457 × 3 = 7371	5374 × 6 = 32244	3657 × 8 = 29256

11 RIGHT OUT OF 12 SCORE 9 LINE L

How to Improve Your Score in Multiplication. If your score on the above test is below 9, it is probably due to one of the following: (1) an imperfect mastery of the 100 multiplication facts, which may be corrected by studying Remedial Sheet 1 on page 145, or (2) difficulty in carrying in multiplication, which may be corrected as explained below.

69 Multiplicand
8 Multiplier
552 Product

In multiplying the numbers at the right, think as follows: "8 × 9 = 72." Write 2 and remember the 7 which is carried in the next step. Then think "8 × 6 = 48. 48 + 7(carried) = 55." Write 55. The product is 552. In this work the addition of 48 and 7 is done *mentally*, hence carrying in multiplication requires the ability to add mentally a one-figure number to a two-figure number. You can improve your ability to do this mental addition by practicing the work on adding by endings on Remedial Sheet 2 on page 146. You found on page 2 that skill in adding by endings is also needed in column addition.

In multiplication the number that is multiplied is called the *multiplicand;* th number by which you multiply is called the *multiplier;* and the answer is called t *product.* Numbers that are multiplied together are called *factors.*

(1) Do Improvement Test A2 on Addition. See page 141.

If you wish to multiply 867 by 243, the steps are as follows:

Multiply 867 by 3 which gives 2601. Write 2601 so that the 1 is directly under the 3 of 243. Next multiply 867 by 4 which gives 3468. Write 3468 so that the 8 is under the 4 of 243. Multiply 867 by 2 which gives 1734. Write 1734 so that the 4 of 1734 is under the 2 of 243.

Add 2601, 3468, and 1734 in the position in which they stand. This gives 210,681 which is the product of 867 by 243. The work should be checked by one of the methods explained below.

```
  867
  243
 2601
3468
1734
210681
```

In the above example, the numbers 2601, 3468, and 1734 are called **partial products.**

The right-hand figure of each partial product must be placed directly under the figure by which you are multiplying.

Checking Multiplication. Multiplication can be checked by one of these methods:

1. *Going over the Work Again.* When the multiplier is a one-figure number, the usual method of checking multiplication is to go over the work mentally, repeating each step of the original multiplication. This method may also be used when the multiplier has two or more figures provided every step of the work is repeated.

2. *Reversing the Numbers.* This method is most conveniently applied when both the multiplicand and the multiplier have about the same number of figures, as in the example at the top of the page where 867 is multiplied by 243. If these numbers are reversed in position and then multiplied, as shown at the right, the product is 210,681 which is the same as the product obtained at the top of the page; hence the work is considered correct. If the products are not alike, some mistake has been made.

```
Check
  243
  867
 1701
1458
1944
210681
```

3. *Casting out 9's.* This method, explained on page 8, is the shortest check for multiplication when the multiplier has two or more figures.

Copy and multiply. Check by one of the methods explained above:

(2)	69 × 607	95 × 710	27 × 134	45 × 7401	78 × 9084
(3)	45 × 438	78 × 758	94 × 135	27 × 8460	67 × 5284
(4)	78 × 378	36 × 607	71 × 208	89 × 1890	79 × 5067
(5)	936 × 890	375 × 508	543 × 967	912 × 7812	758 × 8102
(6)	542 × 679	689 × 470	925 × 495	247 × 2075	875 × 6934
(7)	327 × 234	954 × 134	378 × 902	366 × 2194	797 × 9015
(8)	198 × 285	168 × 513	567 × 237	435 × 5712	386 × 5678

For more practice, see rows 1 to 9 on page 152.

CASTING OUT 9's

The check of 9's, which has been in use for over a thousand years, is the shortest method of checking multiplication when the multiplier contains two or more figures. This check is also known as "casting out 9's."

Suppose you have multiplied 296 by 626 and wish to check the work to see if the product, 185,296, is correct. This may be done by casting out 9's as follows:

	9's check
296	8
626	5
1776	40 (4)
592	
1776	
185296 (4)	

1. Add the digits of 296, thus: $2 + 9 + 6 = 17$. Since 17 is larger than 9, add the digits of 17, thus: $1 + 7 = 8$. 8 is the *check number* for 296.

2. Add the digits of 626, thus: $6 + 2 + 6 = 14$; then add the digits of 14 which gives $1 + 4 = 5$. 5 is the *check number* for 626.

3. Multiply the check number 8 by the check number 5, which gives 40. Add the digits of 40 which gives $4 + 0$ or 4. Write 4 in parentheses. This 4 is called the *final* check number.

4. Add the digits of 185,296, thus: $1 + 8 + 5 + 2 + 9 + 6 = 31$; then add the digits of 31, thus: $3 + 1 = 4$. 4 is the check number for 185,296. This 4 is the same as the final check number 4 obtained in Step 3. Hence the work is considered correct.

5. When you add the digits of a number, you may save time by skipping a 9 or any digits whose sum is 9. For example, in adding the digits of 296, you may skip the 9 and add only 2 and 6 which gives 8 as a check number. Likewise, in 185,296, you may skip 1 and 8 and also the 9, adding only 5, 2, and 6 which gives 4 as a check number.

When the check number of the product is the same as the final check number, the work is considered correct; see Steps 3 and 4 above. When the check number of the product differs from the final check number, the work is wrong; hence you should go over the work to find the mistake. In example *A* below, the work is wrong because the final check number 3 differs from the check number 2. Find the mistake in *A* and check again.

A		B		C	
554	5	429	6	438	6
168	6	174	3	205	7
4432	30 (3)	1716	18 (0)	2190	42 (6)
3224		3003		876	
554		429		10950 (6)	
92072 (2)		74646 (0)			

The check number must always be less than 9. If the sum of the digits equals 9 you should "cast out," or drop, the 9, which leaves 0 as the check number. For example, the check numbers for 513, 4851, and 8676 are each 0. Example *B* above is a case where the final check numbers are each 0; since the check numbers are the same, the work is considered right. The check of 9's fails when two figures are transposed, but this rarely happens during the process of computation; it also fails when a partial product is misplaced, as in example *C*, which is discussed on page 9.

Example *C*, given on page 8, is an interesting case. Since the final check numbers are each 6, the work appears to be correct, yet the work contains an error since the partial product, 876, has been written one place too far to the right. The product, 10,950, is wrong in spite of the agreement of the check numbers. *Whenever a partial product is misplaced, the check of 9's will not detect the error, hence it is important to see that each partial product is in the right place before casting out 9's.*

The failure of the 9's check in example *C* on page 8 should not cause one to abandon this check since a rapid inspection of the partial products will show whether or not they are properly placed. The 9's check is by far the quickest check for multiplication; it is more rapid than the usual methods of going over the work or of reversing the factors and multiplying again.

Watching Zeros in Multiplication. In example *D* the multiplier, 506, contains a zero; the work could be done as shown in *D*, writing a row of zeros in the second partial product; it is shorter, however, to omit the row of zeros and to do the work as shown in *E*. In either case be sure to write 1735 so that the 5 of 1735 comes under the 5 of 506.

In example *F*, the multiplier, 730, ends in 0; in this case, let the 0 of 730 stand out at the right; then bring down the 0 of 730 and multiply 374 by 3. Be sure to write the 8 of 2618 under the 7 of 730.

D	E	F
347	347	374
506	506	730
2082	2082	11220
000	1735	2618
1735	175582	273020
175582		

Copy and multiply. Check by reversing the numbers or by casting out 9's:

	a	*b*	*c*	*d*	*e*
(1)	807 × 481	805 × 580	605 × 3941	209 × 3640	408 × 4818
(2)	709 × 834	305 × 718	206 × 3845	604 × 9023	807 × 5601
(3)	309 × 267	709 × 230	109 × 7089	506 × 1230	806 × 4009
(4)	404 × 470	303 × 409	804 × 3249	680 × 2170	907 × 6108
(5)	506 × 657	703 × 356	780 × 4609	605 × 2309	930 × 1246
(6)	760 × 607	860 × 930	890 × 1560	930 × 2360	740 × 3406
(7)	280 × 734	420 × 985	270 × 2890	510 × 1567	220 × 4568
(8)	360 × 458	780 × 830	930 × 7860	750 × 4320	750 × 5512
(9)	480 × 726	508 × 694	640 × 8540	290 × 1986	320 × 7508

For more practice, see rows 10 to 12 on page 152.

SHORT DIVISION

(1) Do Improvement Test A3 on Addition. See page 141.

Successful work in division requires a mastery of the *even division* facts; two of these facts are given at the right. The first fact means that there are five 3's in 15. These are called *even* division facts since the division is exact. These division facts are given on Remedial Sheet 4 on page 148. It is important to practice on these facts before proceeding further.

$$3\overline{)15}\quad 5 \qquad 7\overline{)42}\quad 6$$

You must also have a knowledge of the *uneven division* facts, each of which involves a remainder. Two of these facts are given at the right. The first fact shows that there are five 3's in 17 with 2 left over; that is, there is a remainder of 2. R means remainder. The second fact shows that there are seven 4's in 31 with a remainder of 3. To get the answer to an uneven division fact like $4\overline{)31}$, you must find the *largest number* of 4's that 31 will contain. If you do not see the answer at once, try several numbers as follows. Try 8; 8 is too large because eight 4's are 32 which is larger than 31. Try 7; 7 is all right because seven 4's are 28 which is the *largest* number of 4's below 31. Hence the quotient is 7. To find the remainder, subtract 28 from 31 mentally which gives a remainder of 3. In finding the answers for the uneven division facts, all the work, including the subtraction, should be done mentally. You should test your knowledge of the uneven division facts by going over Remedial Sheet 5 on page 149.

$$3\overline{)17}\quad 5\text{ R}2 \qquad 4\overline{)31}\quad 7\text{ R}3$$

The term "short division" refers to all exercises in division having a one-figure divisor and a quotient of two or more figures such as example *A* at the right. If the exercise has a one-figure quotient, you are dealing with an even or an uneven division fact. In working exercises in short division, expert computers do all the work mentally, writing only the quotient and the remainder. You should do the same, particularly in exercises having divisors of 2, 3, 4, and 5. If you make many mistakes when you do the work mentally in exercises having divisors of 6, 7, 8, and 9, it is better to write out the steps as shown in *B* at the right.

A	B	Check for B
$3\overline{)176}$	$9\overline{)718}$	79
58 R2	79 R7	9
	63	711
	88	7
	81	718
	7	

When an exercise has a remainder, the remainder is sometimes written over the divisor to form a fraction in the quotient. In example *A*, shown above, the answer may be written as 58 R2 or as $58\frac{2}{3}$. In example *B*, 718 is the *dividend*, 9 is the *divisor*, 79 is the *quotient*, and 7 is the *remainder*.

To check example *B*, proceed as follows: Multiply the quotient 79 by the divisor 9 which gives 711. Then add the remainder 7 to 711, which gives 718. Since 718 is equal to the dividend, the work is correct.

To check division, multiply the quotient by the divisor and add the remainder to the product. The result should equal the dividend.

Improvement Test — Subtraction D1 Time 4 min.

	a	*b*	*c*	*d*	*e*
(1)	82715 31856	35072 29994	75699 63884	18289 9296	60054 6897
(2)	54796 48289	11111 6809	79373 41877	40000 19498	33012 7098
(3)	91100 64306	24353 9189	54118 15225	38540 6295	15283 7635

-------- RIGHT OUT OF 15 SCORE -------- LINE N

Division. Divide and check. Express remainders, if any, as fractions in the quotient:

(4)	4)344	2)5351	5)4804	3)13192/3	5)5440
(5)	3)234	4)6359	2)7217	4)3240	3)3483
(6)	4)276	5)2800	3)29403	2)1656	5)3633
(7)	6)270	8)12676	7)2422	9)2115	6)3725
(8)	9)2541	7)5873	9)3348	6)4464	8)2736
(9)	7)3054	6)39415	8)29677	9)36612/9	7)18183
(10)	6)5395	3)10702/3	7)41042/7	8)15598	9)13094

For more practice on subtraction, see page 150.

LONG DIVISION

Suppose you wish to divide 1676 by 31. First select the smallest part of 1676 that will contain 31; you see that 16 does not contain 31 but 167 does contain it. 167 is called the first partial dividend. The quotient figures are found as follows:

Step 1. The first quotient figure tells how many 31's there are in 167; a quick way to *estimate* this quotient figure is to divide 16 by 3 (the first figure of 31), which gives 5. This 5 is a trial quotient figure. You will not know whether it is the correct quotient figure until you complete Step 4 below. Write 5 over 7 of 167. *The first quotient figure must always be written over the last figure of the first partial dividend.*

```
  54
31)1676
  155
  126
  124
    2
```

Step 2. Multiply 31 by 5, which gives 155.

Step 3. Subtract 155 from 167, which gives a remainder of 12.

Step 4. Compare 12 with the divisor 31. Since 12 is less than 31, 5 is the right quotient figure. *When you have the right quotient figure, the remainder will always be less than the divisor.*

Step 5. Bring down 6, which gives 126 as the next partial dividend.

The five steps are now repeated to get the next quotient figure.

Step 1. Divide 12 by 3, which gives 4 as the next trial quotient figure.

Step 2. Multiply 31 by 4, which gives 124.

Step 3. Subtract 124 from 126, which gives a remainder of 2.

Step 4. Compare 2 with 31. Since 2 is less than 31, 4 is the right quotient figure.

The quotient is 54 with a remainder of 2. To check, multiply 54 by 31 and add the remainder, 2; the result should equal the dividend, 1676.

Rule I. *When a two-figure divisor ends in* 1, 2, 3, 4, *or* 5, *divide each partial dividend by the first figure of the divisor to estimate each figure of the quotient.*

Copy and divide. Check by multiplying:

	a	*b*	*c*	*d*	*e*
(1)	7035 ÷ 21	5490 ÷ 45	4745 ÷ 73	2048 ÷ 32	2898 ÷ 63
(2)	2592 ÷ 81	9460 ÷ 55	1443 ÷ 32	4799 ÷ 51	4608 ÷ 64
(3)	9782 ÷ 73	6898 ÷ 55	9339 ÷ 74	5376 ÷ 24	4337 ÷ 92
(4)	3975 ÷ 75	8862 ÷ 42	9968 ÷ 81	3844 ÷ 62	9199 ÷ 22
(5)	7071 ÷ 43	8568 ÷ 34	5493 ÷ 84	3928 ÷ 85	9956 ÷ 44
(6)	5859 ÷ 93	2137 ÷ 83	7921 ÷ 95	7896 ÷ 23	5976 ÷ 85
(7)	1569 ÷ 65	9689 ÷ 72	2391 ÷ 53	5987 ÷ 94	3418 ÷ 71
(8)	9648 ÷ 24	2175 ÷ 62	1794 ÷ 33	4993 ÷ 54	8486 ÷ 35
(9)	4970 ÷ 35	7826 ÷ 25	6338 ÷ 44	3418 ÷ 75	8537 ÷ 53

For more practice, see rows 13 to 23 on page 152.

The check of 9's is the most rapid method of checking long division in examples where the divisor has two or more figures.

Before applying the 9's check to division, it is important to recall that any example in division may be checked by using the following fact:

$$(divisor \times quotient) + remainder = dividend$$

The check of 9's is applied by using the same fact, as follows:

$$\left(\begin{matrix}check\ number\\ of\ divisor\end{matrix} \times \begin{matrix}check\ number\\ of\ quotient\end{matrix}\right) + \begin{matrix}check\ number\\ of\ remainder\end{matrix} = \begin{matrix}check\ number\\ of\ dividend\end{matrix}$$

The use of the 9's check in division is illustrated below.

In the example at the right the divisor, 23, has a check number of 5; the quotient, 456, has a check number of 6; the remainder, 10, has a check number of 1; and the dividend, 10,498, has a check number of 4. These check numbers are obtained by adding the digits as in multiplication.

To check the work, multiply the check number (5) of the divisor by the check number (6) of the quotient which gives 30; then add the check number (1) of the remainder which gives 31. Adding the digits of 31 gives 4 as the final check number. Since the check number of the dividend is also 4, the work is considered correct.

```
          (6)          9's check
(5)  456                   5
23)10498  (4)            6
  92                      30
  129                   + 1
  115                     31  (4)
   148
   138
    10  (1)
```

When the final check number differs from the check number of the dividend, the work is wrong. For example, if you divide 17,881 by 32 and get a quotient of 561 with a remainder of 29, the check of 9's will show that there is a mistake in the work since the final check number does not agree with the check number of the dividend. Check the example just given, and verify this statement.

The check of 9's is recommended only for checking multiplication and division; it is not recommended for checking addition and subtraction since the 9's check has no advantage in time over the usual checks for these operations.

Copy and divide. Check the work by casting out 9's:

	a	*b*	*c*	*d*
(1)	23,698 ÷ 62	23,534 ÷ 32	74,158 ÷ 92	22,973 ÷ 65
(2)	18,978 ÷ 75	39,928 ÷ 84	12,998 ÷ 53	52,978 ÷ 85
(3)	38,321 ÷ 93	13,485 ÷ 53	21,584 ÷ 93	27,789 ÷ 43
(4)	30,660 ÷ 73	18,798 ÷ 44	11,685 ÷ 41	14,498 ÷ 35
(5)	45,795 ÷ 65	37,076 ÷ 92	52,775 ÷ 74	33,385 ÷ 55
(6)	19,376 ÷ 55	39,947 ÷ 81	19,667 ÷ 54	51,738 ÷ 63

For more practice, see rows 13 to 23 on page 152.

(1) Do Improvement Test D2 on Subtraction. See page 142.

In example *A*, the trial quotient figure is obtained by dividing 15 by 5, which gives 3. $3 \times 53 = 159$; 159 cannot be subtracted from 152 since 159 is greater than 152. Hence 3 is too large as a quotient figure.

A. Wrong	B. Right
3	2
53)1528	53)1528
159	106
	46

Try 2 as the first quotient figure, as shown in *B*; 2 is found to be correct since the remainder 46 is less than 53. To get the second quotient figure, bring down 8 to form a new partial dividend of 468; then divide 46 by 5, which gives 9 as a trial quotient figure. Since 9 is too large, try 8 and complete the work.

In a very few examples the trial quotient figure has to be corrected twice before the true quotient figure is found but such cases are rather rare. The example, 190 ÷ 24, illustrates one of these cases. The first trial quotient figure is 9, which is too large; 8 is also too large but 7 is found to be correct. Thus, two corrections are necessary in order to get the correct quotient figure.

If the trial quotient figure gives a product which is larger than the partial dividend, the quotient figure is too large. Try the next smaller figure.

Copy and divide. Check by multiplying or by casting out 9's. If there is a remainder, write it over the divisor to form a fraction in the quotient:

	a	*b*	*c*	*d*	*e*
(2)	2401 ÷ 33	2813 ÷ 75	1200 ÷ 25	12,100 ÷ 35	24,103 ÷ 62
(3)	3063 ÷ 52	4872 ÷ 84	8064 ÷ 43	49,830 ÷ 65	40,701 ÷ 85
(4)	3649 ÷ 41	2590 ÷ 35	3029 ÷ 34	18,216 ÷ 24	21,283 ÷ 44
(5)	3627 ÷ 93	1568 ÷ 32	2436 ÷ 42	12,206 ÷ 34	21,754 ÷ 73
(6)	5106 ÷ 74	9025 ÷ 33	8712 ÷ 24	28,861 ÷ 73	15,264 ÷ 53
(7)	3276 ÷ 84	6730 ÷ 23	8000 ÷ 23	16,320 ÷ 34	24,667 ÷ 43
(8)	5644 ÷ 83	3717 ÷ 63	4510 ÷ 92	10,028 ÷ 23	40,338 ÷ 83
(9)	7358 ÷ 93	2633 ÷ 53	4248 ÷ 64	18,414 ÷ 62	16,340 ÷ 43
(10)	6090 ÷ 21	2465 ÷ 85	8127 ÷ 43	35,510 ÷ 53	24,667 ÷ 43
(11)	9125 ÷ 25	2418 ÷ 31	1034 ÷ 22	20,350 ÷ 55	30,000 ÷ 52
(12)	8910 ÷ 25	5682 ÷ 95	7942 ÷ 75	13,072 ÷ 44	20,238 ÷ 35
(13)	5931 ÷ 23	1996 ÷ 54	4687 ÷ 94	38,274 ÷ 55	37,307 ÷ 75
(14)	2408 ÷ 35	3802 ÷ 44	2876 ÷ 72	18,027 ÷ 92	25,082 ÷ 63

For more practice, see rows 1 to 5 on page 153.

(1) Do Improvement Test D3 on Subtraction. See page 142.

Add

(2)
3461
1025
2167
1323
5172
9466
5947
5574

Divide

(3) 615
34)20910
204
351
34
170
170
00

Multiply

(4) 86214
437
603498
258642
344856
37675518

Subtract

(5) 12648
11972
676

Subtract

(6) 70000
28019
41981

-------- RIGHT OUT OF 5 SCORE -------- LINE D

ADDING DOLLARS AND CENTS

Add upward. Check by adding downward:

	a	b	c	d	e
(7)	$38.25	$77.64	$49.20	$67.63	$82.93
	60.52	76.40	83.39	5.44	9.60
	35.34	51.98	22.95	22.52	13.91
	71.46	26.81	58.40	56.15	4.79
	42.75	62.15	70.89	71.63	90.47
	13.22	37.87	99.08	53.28	43.88
	27.54	62.24	15.11	9.56	78.16
	34.36	35.65	63.98	67.05	83.40
	56.73	90.74	37.71	84.89	14.74
	43.08	43.52	30.37	43.40	23.24
	$399.25	$565.00	$530.88	$482.55	$445.17
(8)	$14.36	$20.72	$13.66	$57.62	$38.61
	97.92	73.25	69.75	98.96	97.57
	8.22	51.84	27.79	84.11	40.82
	57.14	95.95	86.42	8.69	7.19
	24.58	21.28	9.08	44.17	84.38
	67.80	90.33	43.65	37.48	56.53
	83.56	77.99	6.58	60.83	9.78
	9.61	24.16	54.44	77.97	48.65
	50.16	13.64	10.50	89.02	76.09
	21.57	52.51	14.07	31.48	25.37
	$424.92	$549.67	$335.94	$560.33	$484.98

In example *A* the trial quotient figure, 6, is obtained by dividing 20 by 3, which gives 6. You find that 6 is too large since 234 is greater than 208. You then try 5, as shown in *B*; 5 is correct since 13 is less than 39. You could avoid these changes and get 5 on the first trial if you should divide 20 by 4 instead of 3. Since 39 is closer to 40 than it is to 30, it is more sensible to estimate the quotient figure by dividing by 4 rather than by 3. Notice that 4 is 1 *more* than the first figure of 39.

A. Wrong	B. Right
6	5
39)208	39)208
234	195
	13

Likewise, when the divisor is 38, or 37, or 36, it is better to estimate the quotient figures by dividing by 4 rather than by 3 since each of these numbers is closer to 40 than to 30. When the divisor is 59, it is better to divide by 6 since 59 is so close to 60. Note that 6 is 1 more than the first figure of 59.

Rule II. *When a two-figure divisor ends in* 6, 7, 8, *or* 9, *divide each partial dividend by* 1 *more than the first figure of the divisor to estimate each quotient figure.*

See the suggestions relative to long division on page 153.

Copy and divide: Check by multiplying or by casting out 9's:

	a	*b*	*c*	*d*	*e*
(1)	3221 ÷ 68	9465 ÷ 76	8365 ÷ 97	12,185 ÷ 78	24,002 ÷ 27
(2)	5614 ÷ 69	2803 ÷ 37	4773 ÷ 88	11,901 ÷ 46	31,783 ÷ 56
(3)	3041 ÷ 86	2931 ÷ 68	3057 ÷ 58	21,672 ÷ 49	25,440 ÷ 76
(4)	4874 ÷ 77	1897 ÷ 59	4434 ÷ 19	39,008 ÷ 57	18,306 ÷ 67
(5)	6094 ÷ 27	9606 ÷ 29	5059 ÷ 66	16,203 ÷ 28	45,482 ÷ 48
(6)	5304 ÷ 49	6547 ÷ 87	2434 ÷ 39	28,003 ÷ 36	48,543 ÷ 86
(7)	2993 ÷ 66	2284 ÷ 99	7258 ÷ 77	21,027 ÷ 56	30,073 ÷ 59
(8)	2770 ÷ 89	2896 ÷ 67	4249 ÷ 57	14,677 ÷ 69	12,954 ÷ 97
(9)	3057 ÷ 58	2736 ÷ 28	6089 ÷ 98	35,996 ÷ 47	15,060 ÷ 29
(10)	4903 ÷ 79	5273 ÷ 19	1599 ÷ 48	18,073 ÷ 38	39,008 ÷ 57
(11)	4502 ÷ 96	7416 ÷ 28	1729 ÷ 37	72,849 ÷ 98	14,347 ÷ 76
(12)	6494 ÷ 48	6571 ÷ 39	6742 ÷ 89	26,162 ÷ 46	69,101 ÷ 56
(13)	5060 ÷ 78	8217 ÷ 47	4603 ÷ 96	16,685 ÷ 99	18,125 ÷ 47
(14)	4001 ÷ 46	5783 ÷ 88	3938 ÷ 87	25,451 ÷ 67	12,575 ÷ 88
(15)	9255 ÷ 38	6048 ÷ 79	1645 ÷ 36	10,231 ÷ 27	29,126 ÷ 39

For more practice, see rows 6 to 12 on page 153.

Improvement Test — Multiplication G1 Time 4 min.

	a	*b*	*c*
(1)	473 × 356 = 168388	912 × 467 = 425904	837 × 721 = 603477
(2)	428 × 195 = 83460	897 × 894 = 801918	956 × 238 = 227528

3 RIGHT OUT OF 6 SCORE 5 LINE E

Review of Division. Copy, divide, and check. For divisors ending in 1 to 5, use Rule I on page 12 to estimate quotient figures; for divisors ending in 6 to 9, use Rule II on page 16:

	a	*b*	*c*	*d*	*e*
(3)	3710 ÷ 53	2389 ÷ 33	6363 ÷ 27	16,000 ÷ 45	18,124 ÷ 57
(4)	5400 ÷ 75	6025 ÷ 22	5238 ÷ 62	10,346 ÷ 44	20,000 ÷ 41
(5)	5312 ÷ 64	8004 ÷ 23	3361 ÷ 54	18,675 ÷ 45	28,040 ÷ 36
(6)	5971 ÷ 71	3029 ÷ 34	2182 ÷ 94	56,242 ÷ 61	30,501 ÷ 59
(7)	1489 ÷ 35	9072 ÷ 24	7760 ÷ 83	17,990 ÷ 35	18,381 ÷ 28
(8)	1903 ÷ 52	9858 ÷ 53	1669 ÷ 79	46,880 ÷ 65	24,002 ÷ 98
(9)	2533 ÷ 82	6732 ÷ 34	3029 ÷ 39	44,030 ÷ 85	45,700 ÷ 52
(10)	2779 ÷ 44	2254 ÷ 23	4434 ÷ 19	19,126 ÷ 81	75,182 ÷ 86
(11)	2574 ÷ 38	7370 ÷ 59	3201 ÷ 36	19,738 ÷ 71	16,677 ÷ 19
(12)	8346 ÷ 47	9054 ÷ 68	5821 ÷ 87	34,146 ÷ 42	13,529 ÷ 83

For more practice, see page 152 and rows 1 to 12 on page 153.

WATCHING THE REMAINDER IN DIVISION

In using Rule II with two-figure divisors that end in 6, 7, 8, and 9, the correct quotient figure is found on the first trial in 80% of all cases involving these divisors. In the remaining 20% of the cases, Rule II gives a trial quotient that is too small by 1. In these cases the trial quotient figure is corrected by making it 1 larger. The safe way to know whether the quotient figure is correct is to compare the remainder with the divisor, as shown below.

In example *A*, the quotient figure 5 is obtained by using Rule II, that is, by dividing 23 by 4. Since the remainder 42 is larger than 39, 42 contains 39 one more time. Hence 5 is too small as a quotient figure. This means that 237 contains 39 one more time, or 6 times, as shown in *B*. In *B*, the remainder 3 is less than 39, hence 6 is the correct quotient figure.

A	B
5	6
39)237	39)237
195	234
42	3

If the remainder is less than the divisor, the quotient figure is correct. If the remainder is equal to or larger than the divisor, make the quotient figure 1 larger.

Correcting Quotient Figures. Copy, divide, and check. Watch the remainders:

	a	*b*	*c*	*d*	*e*
(1)	4422 ÷ 48	2678 ÷ 29	1976 ÷ 37	28,497 ÷ 69	53,694 ÷ 57
(2)	2791 ÷ 68	5914 ÷ 27	8881 ÷ 26	53,856 ÷ 88	47,348 ÷ 76
(3)	8099 ÷ 88	3128 ÷ 46	5184 ÷ 96	14,094 ÷ 27	35,796 ÷ 38
(4)	2686 ÷ 79	6264 ÷ 87	5592 ÷ 69	23,870 ÷ 77	80,997 ÷ 87
(5)	5776 ÷ 19	7771 ÷ 19	4902 ÷ 57	17,591 ÷ 49	26,096 ÷ 28
(6)	9108 ÷ 99	2397 ÷ 26	3060 ÷ 68	41,205 ÷ 67	15,281 ÷ 37
(7)	1751 ÷ 56	2166 ÷ 38	7310 ÷ 86	41,760 ÷ 58	63,987 ÷ 77
(8)	1320 ÷ 28	2784 ÷ 58	3072 ÷ 48	26,136 ÷ 36	53,368 ÷ 56
(9)	6403 ÷ 19	6179 ÷ 76	2412 ÷ 67	25,774 ÷ 49	27,612 ÷ 39
(10)	3128 ÷ 76	2124 ÷ 36	5629 ÷ 29	27,324 ÷ 66	35,577 ÷ 59
(11)	3487 ÷ 26	5796 ÷ 86	3652 ÷ 59	19,458 ÷ 47	12,594 ÷ 26
(12)	1986 ÷ 47	9747 ÷ 68	6399 ÷ 79	35,792 ÷ 89	39,658 ÷ 79
(13)	1789 ÷ 38	7459 ÷ 98	5544 ÷ 66	18,974 ÷ 28	79,432 ÷ 86
(14)	3996 ÷ 97	5126 ÷ 58	3754 ÷ 87	35,990 ÷ 59	47,897 ÷ 77
(15)	6552 ÷ 78	8427 ÷ 39	2889 ÷ 78	14,953 ÷ 19	20,816 ÷ 48
(16)	4758 ÷ 66	3949 ÷ 47	3894 ÷ 56	21,904 ÷ 37	26,833 ÷ 37
(17)	8892 ÷ 26	5795 ÷ 89	5492 ÷ 96	41,457 ÷ 98	34,931 ÷ 69

For more practice, see rows 13 to 18 on page 153.

(1) Do Improvement Test G2 on Multiplication. See page 144.

Quotient Figures by Inspection. A rapid inspection of the divisor and the partial dividend should always be made to see if the quotient figure is immediately evident. If so, the quotient figure thus obtained should be used without applying any rule. For example, if the divisor is 46 and the first partial dividend is 49, as shown in the example at the right, it is immediately evident that the first quotient figure is 1. It is also evident that the second quotient figure is 0 since 35 does not contain 46.

```
  10
46)4957
  46
  35
```

Troublesome Divisors. The divisors 13, 14, 15, 16, 17, and 18 are six troublesome divisors to which Rules I and II do not apply. When you have one of these divisors, mentally estimate the quotient figure as well as you can and then multiply the divisor by the estimated figure to see if that figure is correct. To divide by 19, however, divide by 2 to estimate the quotient figure.

Three-Figure Divisors. The rules for handling three-figure divisors are similar to Rules I and II. If the *second figure* of the three-figure divisor is 1, 2, 3, 4, or 5, divide by the first figure of the divisor to estimate the quotient figures. If the second figure is 6, 7, 8, or 9, divide by 1 more than the first figure.

For example, if the divisor is 528, divide by 5 since the second figure is 2. If the divisor is 578, divide by 6 since the second figure is 7.

Copy, divide, and check:

	a	*b*	*c*	*d*
(2)	10,565 ÷ 15	10,039 ÷ 16	12,240 ÷ 17	10,322 ÷ 13
(3)	16,427 ÷ 18	11,654 ÷ 13	12,128 ÷ 16	12,036 ÷ 17
(4)	12,662 ÷ 13	13,009 ÷ 17	16,254 ÷ 18	11,662 ÷ 14
(5)	12,838 ÷ 14	12,540 ÷ 15	11,295 ÷ 14	11,286 ÷ 18
(6)	29,148 ÷ 27	39,300 ÷ 35	10,127 ÷ 97	68,315 ÷ 64
(7)	41,238 ÷ 711	15,645 ÷ 745	40,294 ÷ 693	424,416 ÷ 567
(8)	28,668 ÷ 391	37,257 ÷ 663	53,132 ÷ 718	178,120 ÷ 292
(9)	10,129 ÷ 267	56,862 ÷ 729	13,786 ÷ 226	561,947 ÷ 799
(10)	64,449 ÷ 837	40,012 ÷ 929	16,542 ÷ 459	105,540 ÷ 196
(11)	34,932 ÷ 492	73,159 ÷ 785	34,937 ÷ 483	118,272 ÷ 231
(12)	18,816 ÷ 336	38,198 ÷ 538	63,423 ÷ 729	200,431 ÷ 375
(13)	59,670 ÷ 918	40,138 ÷ 658	11,671 ÷ 243	221,260 ÷ 851

1. Mr. Adams drove his car 235 mi. on Monday, 284 mi. on Tuesday, 310 mi. on Wednesday, and 319 mi. on Thursday. Find the average distance Mr. Adams drove per day.

To find the average, add the four numbers and divide the sum by 4. The average per day is 287 mi. Mr. Adams drove a different distance each day, his total distance being 1148 mi. for the 4 days. If he had driven the average distance of 287 mi. each day, his total distance would also have been 1148 mi. The average tells what would have happened if Mr. Adams had covered 1148 mi. but had driven the *same* distance each day.

$$\begin{array}{r} 235 \\ 284 \\ 310 \\ 319 \\ \hline 4)\overline{1148} \\ 287 \end{array}$$

2. Mr. Smith drove his car a total of 1437 mi. in 5 days. How many miles per day did he average?

$$\begin{array}{r} 5)\overline{1437} \\ 287\frac{2}{5} \end{array}$$

There are no numbers to be added in this problem because the total distance is given. The average distance per day is $287\frac{2}{5}$ mi. If Mr. Smith had driven $287\frac{2}{5}$ mi. each day for 5 days, his total distance would also have been 1437 mi.

3. John keeps chickens. On Monday he gathered 43 eggs; on Tuesday, 41 eggs; on Wednesday, 47 eggs; on Thursday, 45 eggs; on Friday, 44 eggs; on Saturday, 42 eggs. Find the average number of eggs he gathered per day.

The answer is $43\frac{2}{3}$ eggs. This answer doesn't seem sensible because it is impossible to gather $\frac{2}{3}$ of an egg each day. In this case, you can say that the average is *about* 44 eggs a day.

4. The weights in pounds of the boys on the high school baseball team are as follows: 135, 141, 125, 128, 140, 148, 150, 147, 137. Find the average weight of the boys on the baseball team.

5. Last month Tom's marks in his high school subjects were as follows: algebra, 76; English, 82; science, 90; social studies, 70; Latin, 94; drawing, 92. What was his average mark last month?

6. Joe had two tests in arithmetic. On the first test his mark was 76; on the second test his mark was 90. What was his average mark on the two tests?

7. Alice got a mark of 70 on an arithmetic test. She will have another test next week. What mark will she have to get on the second test to bring her average mark on the two tests up to 80?

Copy and find the average of the numbers in each exercise:

8. 27, 35, 40, 47, 50, 38, 29.

9. 216, 240, 223, 229, 245, 223.

10. 63, 65, 72, 60, 58, 67, 69, 71, 78.

11. 649, 613, 637, 680, 605, 641, 643, 650.

12. 1267, 1240, 1259, 1288, 1300, 1350, 1220, 1271, 1275, 1270.

DIAGNOSTIC TEST

(1) Do Improvement Test G3 on Multiplication. See page 144.

Subtract ex. a and b:

(2)

a

34708
19156
15552

b

900000
892754
7246

Divide ex. c and d:

c

464
3)1392

d

2943
8)23552

Add ex. a and b:

(3)

a

7993
6787
8791
6629
9902
6861
7691
8803
1123
64580

b

$65.13
96.89
26.31
45.24
51.24
58.58
37.66
20.90
72.70
$470.54

Multiply ex. c and d:

c

756
684
3014
5948
453.6
516104

d

2417
390
217530
7251
942630

Divide the following:

(4)

53
43)2279
215
129
129
00

914
47)19458
188
65
47
188
188
00

69
72)4968
432
648
648
00

5257
89)46782
445
228
178
502
445
57
89

17 RIGHT OUT OF 12

SCORE 18 LINE L

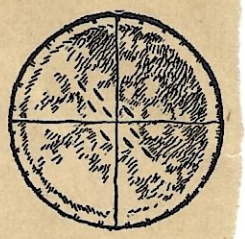
REMEDIAL EXERCISES

See page 138.

Ex. 2 *a*	p. 150, 1–6	Ex. 3 *a*	p. 151, 1–4	Ex. 4 *a*	p. 152, 13–23
Ex. 2 *b*	p. 150, 7–8	Ex. 3 *b*	p. 151, 1–4	Ex. 4 *b*	p. 153, 13–18
Ex. 2 *c*	p. 11, 4–6	Ex. 3 *c*	p. 152, 1–9	Ex. 4 *c*	p. 153, 1–5
Ex. 2 *d*	p. 11, 7–10	Ex. 3 *d*	p. 152, 10–12	Ex. 4 *d*	p. 153, 6–12

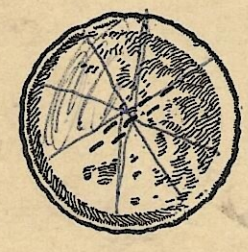
STUDYING FRACTIONS

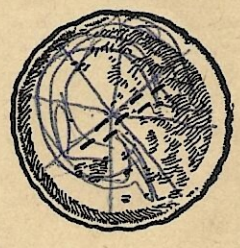
1. This pie is cut into 4 equal pieces. Each piece is called one quarter of the pie. Another name that means *one fourth* is one fourth. Shade 3 fourths of this pie.



2. Write 3 fourths using numbers. $\frac{3}{4}$ If you eat $\frac{1}{4}$ of the pie, you will have $\frac{3}{4}$ of the pie left.

3. By drawing lines through the center of each pie below, divide each pie into 8 equal parts. Shade $\frac{3}{8}$ of the first pie. Shade $\frac{5}{8}$ of the second pie. Shade $\frac{7}{8}$ of the third pie. What part of the third pie is not shaded? ------





In the above picture, you shaded $\frac{7}{8}$ of the third pie. In the fraction $\frac{7}{8}$, 7 is the *numerator* and 8 is the *denominator*. 7 and 8 are called the *terms* of the fraction. The denominator 8 shows that 1 whole thing, like a pie, has been divided into 8 *equal* parts. The numerator 7 shows that 7 of these parts have been taken.

$\frac{7}{8}$ = Numerator
= Denominator

4. There are 15 stars at the right. Draw a line around $\frac{2}{3}$ of these stars. $\frac{2}{3}$ of 15 stars are 10 stars.

5. There are 16 plants in the row below.

Divide the row of plants into *eighths* by drawing lines around every 2 plants. In this exercise you have divided a *group* of 16 plants into 8 equal parts; each of these equal parts contains 2 plants. $\frac{3}{8}$ of all the plants are 6 plants. Draw a heavy line under $\frac{5}{8}$ of all the plants. $\frac{5}{8}$ of all the plants are 10 plants.

Find the following:

	a	*b*	*c*	*d*
6.	$\frac{1}{8}$ of 24 = 3	$\frac{1}{5}$ of 25 = 5	$\frac{1}{7}$ of 28 = 4	$\frac{1}{9}$ of 45 = 5
	$\frac{3}{8}$ of 24 = 9	$\frac{3}{5}$ of 25 = 15	$\frac{2}{7}$ of 28 = 8	$\frac{5}{9}$ of 45 = 25
	$\frac{5}{8}$ of 24 = 15	$\frac{4}{5}$ of 25 = 20	$\frac{6}{7}$ of 28 = 24	$\frac{7}{9}$ of 45 = 35

Improvement Test — Division R1 Time 4 min.

	a	*b*	*c*
(1)	64)23488 → 367; 192; 428; 384; 448; 448; 00	49)42777 → 873; 392; 357; 343; 147; 147; 00	85)37060 → 436; 340; 306; 255; 510; 510; 00
(2)	77)40656 → 528; 385; 215; 154; 616; 616; 00	31)28644 → 924; 279; 774; 624; 1524; 1524; 00	92)12604 → 137; 92; 340; 276; 644; 644; 00

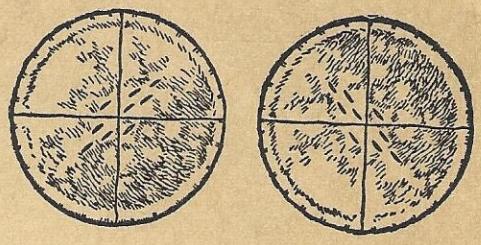
6 RIGHT OUT OF 6 SCORE 10 LINE E

Fractional Parts. Put the right numbers in the spaces. Do the work mentally. In ex. 3 a, think: $\frac{1}{3}$ of 15 = 5, hence $\frac{2}{3}$ of 15 = 2 × 5, or 10:

	a	*b*	*c*	*d*
(3)	$\frac{2}{3}$ of 15 = 10	$\frac{5}{6}$ of 24 = 20	$\frac{2}{9}$ of 18 = 4	$\frac{7}{8}$ of 72 = 63
(4)	$\frac{3}{4}$ of 12 = 9	$\frac{4}{5}$ of 10 = 8	$\frac{3}{5}$ of 40 = 27	$\frac{4}{7}$ of 49 = 28
(5)	$\frac{4}{5}$ of 50 = 40	$\frac{7}{8}$ of 24 = 21	$\frac{2}{3}$ of 21 = 14	$\frac{3}{8}$ of 40 = 15
(6)	$\frac{5}{6}$ of 36 = 30	$\frac{2}{3}$ of 18 = 12	$\frac{3}{5}$ of 25 = 15	$\frac{5}{8}$ of 72 = 45
(7)	$\frac{3}{8}$ of 32 = 12	$\frac{3}{4}$ of 20 = 15	$\frac{5}{6}$ of 48 = 40	$\frac{4}{5}$ of 30 = 24
(8)	$\frac{2}{5}$ of 45 = 18	$\frac{7}{8}$ of 64 = 56	$\frac{3}{5}$ of 10 = 6	$\frac{3}{4}$ of 36 = 27

REDUCING FRACTIONS

A fraction like $\frac{3}{4}$ is called a *proper* fraction because the numerator 3 is smaller than the denominator 4. Fractions like $\frac{5}{4}$ or $\frac{4}{4}$ are called *improper* fractions because the numerator is larger than, or equal to, the denominator. A number like $2\frac{1}{2}$, consisting of a whole number and a fraction, is a *mixed number.*



These pies are divided into _____. Show that $\frac{5}{4}$ of a pie is the same as all of the first pie and $\frac{1}{4}$ of the second pie. This shows that $\frac{5}{4} = 1\frac{1}{4}$. Also show that $\frac{8}{4}$ of a pie is the same as 2 whole pies. This shows that $\frac{8}{4} = 2$.

To change improper fractions like $\frac{5}{4}$ and $\frac{8}{4}$ to mixed numbers or whole numbers, divide the numerator by the denominator.

Reduction of Fractions. Reduce to whole numbers or mixed numbers:

	a	*b*	*c*	*d*	*e*	*f*
(1)	$\frac{9}{4} = 2\frac{1}{4}$	$\frac{9}{3} = 3$	$\frac{27}{8} = 3\frac{3}{8}$	$\frac{20}{9} = 2\frac{2}{9}$	$\frac{23}{5} = 4\frac{3}{5}$	$\frac{47}{6} = 7\frac{5}{6}$
(2)	$\frac{8}{3} = 2\frac{2}{3}$	$\frac{7}{4} = 1\frac{3}{4}$	$\frac{16}{16} = 1$	$\frac{45}{7} = 6\frac{3}{7}$	$\frac{31}{10} = 3\frac{1}{10}$	$\frac{21}{5} = 4\frac{1}{5}$
(3)	$\frac{7}{6} = 1\frac{1}{6}$	$\frac{8}{5} = 1\frac{3}{5}$	$\frac{20}{3} = 6\frac{2}{3}$	$\frac{39}{8} = 4\frac{7}{8}$	$\frac{25}{5} = 5$	$\frac{50}{9} = 5\frac{5}{9}$
(4)	$\frac{6}{6} = 1$	$\frac{9}{5} = 1\frac{4}{5}$	$\frac{19}{12} = 1\frac{7}{12}$	$\frac{33}{8} = 4\frac{1}{8}$	$\frac{40}{5} = 8$	$\frac{49}{9} = 5\frac{4}{9}$
(5)	$\frac{5}{2} = 2\frac{1}{2}$	$\frac{11}{6} = 1\frac{5}{6}$	$\frac{25}{8} = 3\frac{1}{8}$	$\frac{30}{6} = 5$	$\frac{19}{4} = 4\frac{3}{4}$	$\frac{26}{3} = 6\frac{2}{3}$

ADDING FRACTIONS

Mrs. Allen bought 2 chickens at the market. One weighed $3\frac{3}{4}$ lb. and the other weighed $4\frac{1}{4}$ lb. How many pounds of chicken did she buy?

Fractions that have like denominators, like $\frac{3}{4}$ and $\frac{1}{4}$, are added by adding their numerators. Thus $\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$. Hence $3\frac{3}{4} + 4\frac{1}{4} = 7\frac{4}{4}$. Since $\frac{4}{4} = 1$, $7\frac{4}{4}$ is the same as $7 + 1$ or 8. Mrs. Allen bought 8 lb. of chicken. Check the work by going over it.

$$\begin{array}{r} 3\frac{3}{4} \\ 4\frac{1}{4} \\ \hline 7\frac{4}{4} = 8 \end{array}$$

ADDING FRACTIONS

(1) Do Improvement Test R2 on Division. See page 144.

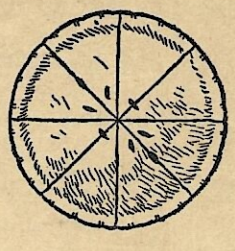
Add the following. Check by going over the work again:

	a	*b*	*c*	*d*	*e*	*f*
(2)	$4\frac{1}{2}$ $3\frac{1}{2}$ 8	$2\frac{3}{4}$ $2\frac{1}{4}$ 5	$7\frac{1}{6}$ $3\frac{5}{6}$ 11	$6\frac{2}{3}$ $8\frac{2}{3}$ 15	$\frac{11}{16}$ $\frac{5}{16}$ 1	$2\frac{1}{5}$ $6\frac{1}{5}$ $8\frac{2}{5}$
(3)	$1\frac{1}{5}$ $7\frac{4}{5}$ 9	$1\frac{2}{3}$ $1\frac{1}{3}$ 3	$2\frac{1}{2}$ $1\frac{1}{2}$ 4	$4\frac{5}{6}$ $5\frac{1}{6}$ 10	$\frac{5}{12}$ $\frac{7}{12}$ 1	$4\frac{1}{3}$ 4 $8\frac{1}{3}$
(4)	$5\frac{5}{6}$ $2\frac{1}{6}$ 8	$3\frac{3}{5}$ $6\frac{3}{5}$ $10\frac{1}{5}$	$1\frac{2}{3}$ $1\frac{2}{3}$ $3\frac{1}{3}$	$7\frac{9}{16}$ $4\frac{7}{16}$ 12	$\frac{4}{5}$ $\frac{4}{5}$ $1\frac{3}{5}$	$8\frac{2}{5}$ $4\frac{2}{5}$ $12\frac{4}{5}$
(5)	$3\frac{2}{3}$ $5\frac{2}{3}$ $9\frac{1}{3}$	$4\frac{3}{5}$ $4\frac{1}{5}$ $8\frac{4}{5}$	$2\frac{1}{2}$ $7\frac{1}{2}$ 10	$8\frac{1}{5}$ $3\frac{1}{5}$ $11\frac{2}{5}$	$\frac{3}{8}$ $\frac{5}{8}$ 1	6 $2\frac{2}{5}$ $8\frac{2}{5}$
(6)	$2\frac{3}{10}$ $4\frac{7}{10}$ 7	$5\frac{7}{12}$ $3\frac{5}{12}$ 9	$1\frac{7}{16}$ $2\frac{9}{16}$ 4	$3\frac{1}{8}$ $2\frac{7}{8}$ 6	$\frac{9}{10}$ $\frac{1}{10}$ 1	$3\frac{2}{3}$ $6\frac{2}{3}$ $10\frac{1}{3}$
(7)	$1\frac{1}{16}$ $2\frac{11}{16}$ $3\frac{5}{16}$ $7\frac{1}{16}$	$2\frac{3}{4}$ $6\frac{1}{4}$ $1\frac{3}{4}$ $10\frac{3}{4}$	$3\frac{1}{6}$ $2\frac{5}{6}$ $5\frac{5}{6}$ $11\frac{3}{6}$	$2\frac{7}{8}$ $1\frac{1}{8}$ $4\frac{5}{8}$ $8\frac{3}{8}$	$\frac{5}{12}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{11}{12}$	$1\frac{1}{8}$ $7\frac{1}{8}$ $4\frac{1}{8}$ $12\frac{3}{8}$

For more practice, see row 1 on page 154.

AN IMPORTANT PRINCIPLE

Shade the upper half of the pie. Then show that $\frac{4}{8}$ of the pie equals $\frac{1}{2}$ of the pie. That is, $\frac{4}{8} = \frac{1}{2}$. When you change $\frac{4}{8}$ to $\frac{1}{2}$, you reduce $\frac{4}{8}$ to lowest terms. A quick way to reduce $\frac{4}{8}$ to $\frac{1}{2}$ is to divide both terms of $\frac{4}{8}$ by 4. Thus $4 \div 4 = 1$, and $8 \div 4 = 2$. Similarly, $\frac{6}{8}$ is reduced to $\frac{3}{4}$ by dividing both 6 and 8 by 2.



If you divide both terms of a fraction by the same number, you get another fraction having the same value.

Reducing Fractions. Reduce each of these fractions to lowest terms:

	a	*b*	*c*	*d*	*e*	*f*
(1)	$\frac{6}{8} = \frac{3}{4}$	$\frac{2}{4} = \frac{1}{2}$	$\frac{10}{12} = \frac{5}{6}$	$\frac{12}{20} = \frac{6}{10} \frac{3}{5}$	$\frac{4}{12} = \frac{1}{3}$	$\frac{14}{16} = \frac{7}{8}$
(2)	$\frac{4}{8} = \frac{1}{2}$	$\frac{2}{8} = \frac{1}{4}$	$\frac{12}{16} = \frac{3}{4}$	$\frac{24}{32} = \frac{3}{4}$	$\frac{6}{10} = \frac{3}{5}$	$\frac{4}{16} = \frac{1}{4}$
(3)	$\frac{4}{6} = \frac{2}{3}$	$\frac{3}{6} = \frac{1}{2}$	$\frac{8}{10} = \frac{4}{5}$	$\frac{6}{16} = \frac{3}{8}$	$\frac{4}{32} = \frac{1}{8}$	$\frac{8}{12} = \frac{2}{3}$

Add. Reduce fractions in sums to lowest terms. Check the work by going over it:

	a	*b*	*c*	*d*	*e*
(4)	$7\frac{1}{8}$ $2\frac{1}{8}$ $9\frac{2}{8} = 9\frac{1}{4}$	$6\frac{1}{8}$ $3\frac{3}{8}$	$4\frac{1}{16}$ $5\frac{5}{16}$	$1\frac{1}{4}$ $8\frac{1}{4}$	$9\frac{5}{8}$ $3\frac{1}{8}$
(5)	$3\frac{3}{4}$ $2\frac{3}{4}$	$5\frac{3}{8}$ $3\frac{3}{8}$	$6\frac{1}{6}$ $4\frac{1}{6}$	$7\frac{7}{8}$ $7\frac{5}{8}$	$3\frac{3}{10}$ $4\frac{3}{10}$
(6)	$4\frac{1}{12}$ $2\frac{5}{12}$	$3\frac{7}{16}$ $1\frac{5}{16}$	$5\frac{5}{6}$ $3\frac{5}{6}$	$2\frac{3}{10}$ $1\frac{9}{10}$	$1\frac{1}{12}$ $5\frac{7}{12}$

For more practice, see row 2 on page 154.

(1) Do Improvement Test R3 on Division. See page 144.

Another Important Principle. On page 26 you learned that you can change $\frac{4}{8}$ to $\frac{1}{2}$ by dividing both 4 and 8 by 4. By reversing this process, you can change $\frac{1}{2}$ back to $\frac{4}{8}$ by multiplying both terms of $\frac{1}{2}$ by 4. In like manner, you can change $\frac{3}{8}$ to $\frac{9}{24}$ by multiplying both terms of $\frac{3}{8}$ by 3. $\frac{3}{8}$ and $\frac{9}{24}$ are alike *in value* but different *in form* just as 4 nickels and 2 dimes are alike in value but different in form.

If you multiply both terms of a fraction by the same number, you get another fraction having the same value.

Write the numerator for the second fraction. In the first example, think: "To change 4 to 8, I multiply 4 by 2; hence I must multiply 3 by 2, which gives 6":

	a	*b*	*c*	*d*	*e*	*f*
(2)	$\frac{3}{4} = \frac{6}{8}$	$\frac{1}{2} = \frac{4}{8}$	$\frac{5}{8} = \frac{10}{16}$	$\frac{5}{6} = \frac{10}{12}$	$\frac{3}{8} = \frac{4}{16}$	$\frac{1}{3} = \frac{2}{6}$
(3)	$\frac{1}{4} = \frac{2}{8}$	$\frac{2}{3} = \frac{4}{6}$	$\frac{1}{4} = \frac{4}{16}$	$\frac{3}{5} = \frac{6}{10}$	$\frac{7}{8} = \frac{14}{16}$	$\frac{1}{2} = \frac{2}{4}$
(4)	$\frac{1}{6} = \frac{2}{12}$	$\frac{1}{2} = \frac{3}{6}$	$\frac{1}{8} = \frac{2}{16}$	$\frac{6}{8} = \frac{12}{16}$	$\frac{1}{2} = \frac{8}{16}$	$\frac{1}{3} = \frac{4}{12}$
(5)	$\frac{2}{4} = \frac{4}{8}$	$\frac{2}{2} = \frac{6}{6}$	$\frac{4}{4} = \frac{16}{16}$	$\frac{5}{6} = \frac{10}{12}$	$\frac{2}{8} = \frac{4}{16}$	$\frac{8}{8} = \frac{16}{16}$
(6)	$\frac{2}{3} = \frac{8}{12}$	$\frac{1}{5} = \frac{2}{10}$	$\frac{5}{8} = \frac{10}{16}$	$\frac{3}{4} = \frac{12}{16}$	$\frac{1}{4} = \frac{3}{12}$	$\frac{2}{4} = \frac{8}{16}$

ADDING FRACTIONS WITH UNLIKE DENOMINATORS

Last Saturday Tom worked for Mr. Allen. In the morning he worked $3\frac{1}{2}$ hr. In the afternoon he worked $2\frac{1}{4}$ hr. How many hours did Tom work in all?

Only fractions with like denominators can be added. To add fractions having different denominators, first change them to fractions having the same denominator. For example, to add $\frac{1}{2}$ and $\frac{1}{4}$, first change $\frac{1}{2}$ to $\frac{2}{4}$. Then add $\frac{2}{4}$ and $\frac{1}{4}$, which gives $\frac{3}{4}$. Tom worked $5\frac{3}{4}$ hr. in all. Check the work by going over it.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{2}{4} \\ 2\frac{1}{4} = 2\frac{1}{4} \\ \hline 5\frac{3}{4} \end{array}$$

To add fractions having different denominators, first change them to fractions having he same denominator.

ADDING FRACTIONS

Add the following. Check by going over the work:

	a	*b*	*c*	*d*
(1)	$3\frac{3}{8}$ + $4\frac{1}{4}$	$2\frac{1}{4}$ + $5\frac{1}{8}$	$5\frac{3}{4}$ + $1\frac{1}{2}$	$3\frac{1}{2}$ + $5\frac{1}{4}$
(2)	$2\frac{7}{8}$ + $2\frac{7}{16}$	$1\frac{5}{8}$ + $4\frac{9}{16}$	$4\frac{5}{8}$ + $5\frac{3}{4}$	$1\frac{1}{16}$ + $1\frac{1}{4}$
(3)	$3\frac{3}{8}$ + $2\frac{1}{2}$	$4\frac{7}{8}$ + $4\frac{1}{4}$	$3\frac{1}{10}$ + $6\frac{2}{5}$	$5\frac{1}{16}$ + $2\frac{3}{4}$
(4)	$6\frac{5}{16}$ + $3\frac{1}{2}$	$3\frac{3}{10}$ + $5\frac{1}{2}$	$4\frac{5}{16}$ + $2\frac{1}{8}$	$3\frac{3}{4}$ + $3\frac{1}{8}$
(5)	$2\frac{1}{3}$ + $5\frac{5}{6}$	$2\frac{3}{4}$ + $1\frac{7}{12}$	$8\frac{5}{6}$ + $6\frac{2}{3}$	$5\frac{5}{12}$ + $3\frac{1}{6}$
(6)	$1\frac{1}{2}$ + $3\frac{2}{3}$ + $2\frac{1}{6}$	$5\frac{1}{4}$ + $2\frac{5}{8}$ + $1\frac{1}{2}$	$3\frac{1}{6}$ + $4\frac{5}{12}$ + $2\frac{3}{4}$	$2\frac{7}{16}$ + $3\frac{1}{4}$ + $5\frac{3}{8}$

For more practice, see row 3 on page 154.

Improvement Test — Addition A4 Time 4 min.

	a	*b*	*c*	*d*	*e*	*f*
(1)	339	227	214	539	452	734
	187	885	950	641	668	256
	460	306	374	274	951	672
	651	298	205	687	186	116
	108	433	727	416	369	497
	556	574	133	168	504	373
	473	228	945	310	925	932
	456	518	363	459	104	248
	5230	3469	3911	3494	4159	3828

6 RIGHT OUT OF 6 SCORE 10 LINE E

MIXED PRACTICE

Put the right numbers in the spaces:

(2) $\frac{5}{6}$ of 18 = 15 $\frac{3}{4}$ of 32 = 24 $\frac{7}{8}$ of 48 = 42 $\frac{3}{8}$ of 72 = 27

Reduce to whole numbers or mixed numbers:

(3) $\frac{9}{8} = 1\frac{1}{8}$ $\frac{6}{6} = 1$ $\frac{6}{4} = 1\frac{2}{4}\ \frac{1}{2}$ $\frac{14}{8} = 1\frac{6}{8}\ \frac{7}{4}$ $\frac{27}{12} = 2\frac{3}{12}\ \frac{1}{4}$

Add:

(4) $8\frac{1}{3}$ $\frac{1}{2}$
$4\frac{1}{6}$
$12\frac{3}{6}\ \frac{1}{2}$

Add:

(5) $12\frac{3}{16}$
$8\frac{5}{16}$
$20\frac{8}{16}\ \frac{1}{2}$

Add:

(6) $4\frac{4}{5}$ $\frac{1}{2}$
$6\frac{7}{10}$
$11\frac{5}{10}\ \frac{1}{2}$

Divide:

(7) $63\overline{)13041}$ 207
126
441

Divide:

(8) $39\overline{)6630}$ 170
39
273

Multiply:

(9) 906
480
0000
7248
3624
434880

15 RIGHT OUT OF 15 SCORE 10 LINE N

ADDING FRACTIONS

Add. Check the work by going over it again:

	a	*b*	*c*	*d*
(1)	$1\frac{1}{2}$ $4\frac{3}{8}$ $3\frac{1}{8}$	$2\frac{1}{4}$ $2\frac{1}{2}$ $2\frac{1}{4}$	$3\frac{5}{16}$ $4\frac{3}{4}$ $1\frac{3}{8}$	$2\frac{1}{10}$ $2\frac{1}{2}$ $6\frac{1}{5}$
(2)	$2\frac{5}{8}$ $5\frac{1}{4}$ $1\frac{1}{2}$	$4\frac{7}{16}$ $1\frac{1}{2}$ $2\frac{5}{8}$	$2\frac{3}{5}$ $1\frac{1}{2}$ $4\frac{9}{10}$	$1\frac{5}{16}$ $1\frac{9}{32}$ $1\frac{1}{2}$
(3)	$4\frac{1}{6}$ $1\frac{1}{2}$ $3\frac{1}{3}$	$6\frac{1}{4}$ $2\frac{2}{3}$ $7\frac{7}{12}$	$1\frac{5}{8}$ $5\frac{1}{16}$ $1\frac{3}{4}$	$6\frac{2}{3}$ $2\frac{11}{12}$ $1\frac{1}{2}$
(4)	$7\frac{7}{8}$ $2\frac{1}{4}$ $3\frac{5}{16}$	$9\frac{7}{10}$ $1\frac{4}{5}$ $4\frac{1}{2}$	$5\frac{3}{4}$ $3\frac{1}{3}$ $5\frac{7}{12}$	$6\frac{5}{6}$ $6\frac{1}{2}$ $3\frac{11}{12}$
(5)	$1\frac{5}{6}$ $1\frac{3}{4}$ $2\frac{1}{12}$ $1\frac{1}{3}$	$2\frac{3}{8}$ $1\frac{9}{16}$ $2\frac{1}{2}$ $2\frac{1}{4}$	$3\frac{1}{2}$ $1\frac{4}{5}$ $2\frac{7}{10}$ $1\frac{3}{5}$	$5\frac{1}{8}$ $2\frac{1}{2}$ $1\frac{5}{8}$ $3\frac{11}{16}$

For more practice, see row 6 on page 154.

(1) Do Improvement Test A2 on Addition. See page 141.

Subtract. To subtract fractions, first see that they have like denominators; then subtract the numerators. Reduce fractions in the answers to lowest terms:

	a	*b*	*c*	*d*	*e*
(2)	$6\frac{7}{8}$ $2\frac{3}{8}$	$8\frac{5}{6}$ $3\frac{1}{6}$	$4\frac{5}{8}$ $4\frac{1}{8}$	$\frac{7}{16}$ $\frac{5}{16}$	$2\frac{1}{2}$ $1\frac{1}{2}$
(3)	$5\frac{3}{4}$ $2\frac{1}{4}$	$2\frac{7}{8}$ $1\frac{3}{8}$	$7\frac{7}{8}$ $3\frac{1}{8}$	$\frac{7}{12}$ $\frac{5}{12}$	$7\frac{3}{4}$ 4
(4)	$9\frac{3}{4}$ $5\frac{3}{4}$	$4\frac{7}{8}$ $1\frac{5}{8}$	$3\frac{7}{10}$ $2\frac{3}{10}$	$\frac{9}{16}$ $\frac{1}{16}$	$4\frac{4}{5}$ $2\frac{2}{5}$
(5)	$8\frac{7}{8} = 8\frac{7}{8}$ $4\frac{1}{2} = 4\frac{4}{8}$	$5\frac{3}{4} = 5\frac{6}{8}$ $3\frac{1}{8} = 3\frac{1}{8}$	$6\frac{3}{4} = 6\frac{6}{8}$ $5\frac{5}{8} = 5\frac{5}{8}$	$\frac{7}{16} = \frac{7}{16}$ $\frac{3}{8} = \frac{6}{16}$	$8\frac{7}{8}$ $6\frac{3}{8}$
(6)	$3\frac{3}{4} = 3\frac{6}{8}$ $2\frac{3}{8} = 2\frac{3}{8}$	$6\frac{1}{4} = 6\frac{2}{8}$ $4\frac{1}{8} = 4\frac{1}{8}$	$5\frac{11}{16} = 5\frac{11}{16}$ $1\frac{3}{8} = 1\frac{6}{16}$	$\frac{3}{8}$ $\frac{6}{16}$ $\frac{3}{16}$ $\frac{3}{16}$	$6\frac{2}{3}$ $3\frac{1}{3}$
(7)	$9\frac{1}{2}$ $\frac{8}{16}$ $3\frac{3}{16}$ $3\frac{3}{16}$	$7\frac{5}{8} = 7\frac{5}{8}$ $5\frac{1}{4} = 5\frac{2}{8}$	$8\frac{3}{5}$ $\frac{6}{10}$ $2\frac{1}{10}$ $\frac{1}{10}$	$\frac{2}{3}$ $\frac{4}{6}$ $\frac{1}{6}$ $\frac{1}{6}$	$5\frac{5}{6}$ $4\frac{1}{6}$

For more practice, see rows 1 and 2 on page 155.

Changing Whole Numbers and Mixed Numbers. If you have \$4, you can then change one of the dollars into 4 quarters. Hence \$4 has the same value as \$3 and 4 quarters; that is, $4 = 3\frac{4}{4}$.

Similarly, if you have $\$4\frac{1}{4}$, you can change one of the dollars into 4 quarters. Hence $\$4\frac{1}{4}$ has the same value as $\$3 + \$\frac{4}{4} + \$\frac{1}{4}$, or $\$3\frac{5}{4}$; that is, $4\frac{1}{4} = 3\frac{5}{4}$. Similarly, you can change $6\frac{5}{8}$ to $5\frac{13}{8}$; in this case you take 1 from 6 and change it to $\frac{8}{8}$; you then add $\frac{8}{8}$ to $\frac{5}{8}$ to make $\frac{13}{8}$. Hence $6\frac{5}{8} = 5\frac{13}{8}$. In subtracting fractions it is often necessary to make changes like those above.

Changing Numbers. Write the numerator of the second fraction:

	a	*b*	*c*	*d*	*e*
(1)	$9 = 8\frac{6}{8}$	$7\frac{1}{2} = 6\frac{13}{2}$	$4\frac{1}{16} = 3\frac{\quad}{16}$	$8\frac{1}{8} = 7\frac{\quad}{8}$	$3\frac{5}{16} = 2\frac{\quad}{16}$
(2)	$6 = 5\frac{\quad}{4}$	$6\frac{1}{4} = 5\frac{\quad}{4}$	$7\frac{3}{4} = 6\frac{\quad}{4}$	$2\frac{3}{8} = 1\frac{\quad}{8}$	$5\frac{1}{3} = 4\frac{\quad}{3}$
(3)	$7 = 6\frac{\quad}{12}$	$9\frac{3}{10} = 8\frac{\quad}{10}$	$5\frac{1}{6} = 4\frac{\quad}{6}$	$3\frac{1}{4} = 2\frac{\quad}{4}$	$4\frac{9}{16} = 3\frac{\quad}{16}$
(4)	$5 = 4\frac{\quad}{8}$	$8\frac{3}{4} = 7\frac{\quad}{4}$	$9\frac{7}{16} = 8\frac{\quad}{16}$	$4\frac{2}{5} = 3\frac{\quad}{5}$	$7\frac{3}{16} = 6\frac{\quad}{16}$

In the following exercises, write the numerators of the fractions:

	a	*b*	*c*
(5)	$7\frac{1}{2} = 7\frac{4}{8} = 6\frac{\quad}{8}$	$9\frac{1}{4} = 9\frac{\quad}{16} = 8\frac{\quad}{16}$	$12\frac{1}{2} = 12\frac{\quad}{4} = 11\frac{\quad}{4}$
(6)	$5\frac{3}{4} = 5\frac{\quad}{12} = 4\frac{\quad}{12}$	$3\frac{5}{8} = 3\frac{\quad}{16} = 2\frac{\quad}{16}$	$15\frac{2}{5} = 15\frac{\quad}{10} = 14\frac{\quad}{10}$
(7)	$4\frac{1}{2} = 4\frac{\quad}{6} = 3\frac{\quad}{6}$	$6\frac{1}{2} = 6\frac{\quad}{16} = 5\frac{\quad}{16}$	$14\frac{1}{2} = 14\frac{\quad}{10} = 13\frac{\quad}{10}$
(8)	$6\frac{2}{3} = 6\frac{\quad}{6} = 5\frac{\quad}{6}$	$5\frac{2}{3} = 5\frac{\quad}{12} = 4\frac{\quad}{12}$	$13\frac{1}{2} = 13\frac{\quad}{12} = 12\frac{\quad}{12}$
(9)	$8\frac{3}{4} = 8\frac{\quad}{8} = 7\frac{\quad}{8}$	$7\frac{5}{6} = 7\frac{\quad}{12} = 6\frac{\quad}{12}$	$11\frac{3}{4} = 11\frac{\quad}{16} = 10\frac{\quad}{16}$

(1) Do Improvement Test A3 on Addition. See page 141.

Subtract and check. In ex. 2a the minuend 6 contains no fraction, hence change 6 to $5\frac{2}{2}$. In ex. 3a, you cannot take $\frac{3}{4}$ from $\frac{1}{4}$, hence change $7\frac{1}{4}$ to $6\frac{5}{4}$:

	a	*b*	*c*	*d*
(2)	$6 = 5\frac{2}{2}$ $2\frac{1}{2} = 2\frac{1}{2}$ $3\frac{1}{2}$	10 $3\frac{5}{8}$	12 $4\frac{3}{10}$	16 $7\frac{7}{16}$
(3)	$7\frac{1}{4} = 6\frac{5}{4}$ $2\frac{3}{4} = 2\frac{3}{4}$ $4\frac{2}{4} = 4\frac{1}{2}$	$8\frac{1}{16}$ $4\frac{5}{16}$	$5\frac{1}{8}$ $2\frac{7}{8}$	$12\frac{1}{16}$ $5\frac{3}{16}$
(4)	$9\frac{1}{8}$ $3\frac{3}{8}$	$7\frac{1}{3}$ $4\frac{2}{3}$	$6\frac{1}{6}$ $1\frac{5}{6}$	$11\frac{7}{16}$ $6\frac{15}{16}$
(5)	$4\frac{1}{2}$ $2\frac{3}{4}$	$9\frac{1}{5}$ $4\frac{7}{10}$	$5\frac{7}{8}$ $3\frac{15}{16}$	$8\frac{5}{16}$ $4\frac{3}{4}$
(6)	$7\frac{3}{8}$ $4\frac{1}{2}$	$8\frac{1}{4}$ $3\frac{5}{12}$	$9\frac{5}{8}$ $2\frac{3}{4}$	$2\frac{3}{8}$ $1\frac{9}{16}$
(7)	$5\frac{1}{6}$ $3\frac{1}{2}$	$7\frac{1}{2}$ $1\frac{9}{10}$	$4\frac{1}{6}$ $2\frac{2}{3}$	$9\frac{1}{4}$ $2\frac{5}{16}$

For more practice, see rows 3 to 5 on page 155.

HARDER SUBTRACTION

Subtract. Check the work by going over it again:

	a	*b*	*c*	*d*
(1)	$7\frac{1}{2}$ − $2\frac{5}{8}$	$6\frac{1}{4}$ − $5\frac{3}{8}$	$9\frac{1}{4}$ − $6\frac{1}{2}$	$4\frac{1}{8}$ − $1\frac{3}{16}$
(2)	$8\frac{3}{4}$ − $6\frac{7}{8}$	$5\frac{1}{8}$ − $2\frac{3}{4}$	$6\frac{2}{3}$ − $1\frac{5}{6}$	$17\frac{1}{16}$ − $5\frac{1}{2}$
(3)	$5\frac{5}{12}$ − $2\frac{3}{4}$	$12\frac{15}{16}$ − $4\frac{9}{32}$	$6\frac{1}{2}$ − $2\frac{3}{4}$	$13\frac{1}{10}$ − $12\frac{1}{2}$
(4)	9 − $7\frac{7}{8}$	$11\frac{3}{4}$ − $5\frac{3}{16}$	$7\frac{5}{8}$ − $3\frac{13}{16}$	18 − $12\frac{7}{32}$
(5)	$4\frac{1}{5}$ − $2\frac{3}{10}$	$10\frac{1}{6}$ − $8\frac{1}{2}$	$8\frac{1}{2}$ − $3\frac{13}{32}$	$12\frac{3}{4}$ − $8\frac{7}{16}$
(6)	$6\frac{1}{6}$ − $2\frac{5}{12}$	$12\frac{3}{8}$ − $3\frac{7}{16}$	$9\frac{5}{12}$ − $4\frac{1}{2}$	$20\frac{1}{4}$ − $5\frac{7}{8}$

For more practice, see rows 3 to 5 on page 155.

Improvement Test — Subtraction D 4 Time 4 min.

	a	*b*	*c*	*d*	*e*
(1)	38160 − 23817	48637 − 12914	15000 − 6809	61377 − 24916	62500 − 49273
(2)	90072 − 87166	79218 − 18736	52808 − 16502	20803 − 20364	88173 − 8295
(3)	70001 − 3860	96281 − 34692	80000 − 40062	51214 − 31718	17527 − 12846

-------- RIGHT OUT OF 15 SCORE -------- LINE N

USING FRACTIONS

(4) Jack was weighed at school today. He weighed $75\frac{3}{4}$ lb. The average weight for a boy of Jack's age is 80 lb. How many pounds underweight is Jack?

(5) Mary Smith is $62\frac{1}{2}$ in. tall and Alice Carter is 64 in. tall. How many inches taller is Alice than Mary?

(6) Tom caught three fish. One weighed $3\frac{3}{8}$ lb., another weighed $2\frac{1}{2}$ lb., and the third weighed $1\frac{3}{4}$ lb. How many pounds did the fish weigh all together? After the fish were cleaned their total weight was 7 lb. How much less was the weight of Tom's fish after cleaning?

(7) Arthur practiced on his violin 1 hr. on Monday, $1\frac{1}{4}$ hr. on Tuesday, $\frac{3}{4}$ hr. on Wednesday, $1\frac{1}{4}$ hr. on Thursday, $1\frac{1}{2}$ hr. on Friday, and $\frac{3}{4}$ hr. on Saturday. How many hours did Arthur practice in all?

(8) The four boys on the relay team of the Cort School made the following times: $8\frac{3}{5}$ sec., $7\frac{1}{2}$ sec., $7\frac{3}{10}$ sec., and $7\frac{4}{5}$ sec. Find their total time.

(9) The four boys on the relay team of the Lincoln School made these times: $7\frac{3}{10}$ sec., $7\frac{1}{5}$ sec., $7\frac{1}{2}$ sec., and $7\frac{9}{10}$ sec. What was their total time? By how many seconds did they beat the team of the Cort School?

MIXED PRACTICE

Subtract

(1) $\begin{array}{r} 809020 \\ -\ 450695 \\ \hline \end{array}$

Multiply

(2) $\begin{array}{r} 768 \\ \times\ 903 \\ \hline \end{array}$

Divide

(3) $67\overline{)41205}$

Add

(4) $\begin{array}{r} 2724 \\ 4661 \\ 1329 \\ 3235 \\ 6584 \\ 1630 \\ 2409 \\ 4831 \\ 9672 \\ 1618 \\ \hline \end{array}$

Divide

(5) $4\overline{)14433}$

Write the missing numerators:

(6) $\frac{2}{3} = \frac{\quad}{12}$ $\quad \frac{3}{4} = \frac{\quad}{16}$ $\quad \frac{7}{8} = \frac{\quad}{16}$ $\quad \frac{5}{6} = \frac{\quad}{12}$

Add

(7) $\begin{array}{r} 3\ \frac{2}{5} \\ 6\ \frac{1}{2} \\ 7\ \frac{3}{10} \\ \hline \end{array}$

Add

(8) $\begin{array}{r} 8\ \frac{7}{32} \\ 2\ \frac{1}{2} \\ 5\ \frac{3}{4} \\ \hline \end{array}$

Subtract

(9) $\begin{array}{r} 7\ \frac{3}{4} \\ -\ 1\ \frac{1}{2} \\ \hline \end{array}$

Subtract

(10) $\begin{array}{r} 6\ \frac{7}{12} \\ -\ 3\ \frac{5}{6} \\ \hline \end{array}$

Find these fractional parts:

(11) $\frac{7}{8}$ of 24 = $\quad \frac{2}{3}$ of 21 = $\quad \frac{4}{5}$ of 45 =

Write the answers in the spaces:

(12) The sum of $6\frac{1}{2}$, $2\frac{3}{4}$, $5\frac{1}{8}$, and $1\frac{1}{4}$ is _____.

(13) Which of these mixed numbers is the smallest, $9\frac{2}{3}$, $9\frac{3}{4}$, or $9\frac{7}{12}$? _____

(14) Find the average of these numbers: 17, 22, 30, 16, 25. _____.

(15) The fraction $\frac{80}{96}$ reduced to lowest terms is _____.

-------- RIGHT OUT OF 20

SCORE -------- LINE T

(1) Do Improvement Test D2 on Subtraction. See page 142.

Fractions with like denominators, such as $\frac{3}{8}$ and $\frac{7}{8}$, are said to have a **common denominator**; in this case the common denominator is 8. In adding fractions like $\frac{1}{2}$ and $\frac{5}{8}$, the fractions will have a common denominator, 8, if $\frac{1}{2}$ is changed to $\frac{4}{8}$; in this example only one fraction, $\frac{1}{2}$, has to be changed. In adding fractions like $\frac{3}{4}$ and $\frac{2}{3}$, however, both fractions have to be changed in order to obtain fractions with like denominators; for these two fractions the common denominator is 12, which is the *product* of the denominators 3 and 4. $\frac{3}{4}$ is changed to $\frac{9}{12}$ while $\frac{2}{3}$ is changed to $\frac{8}{12}$. Hence $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12}$ or $1\frac{5}{12}$.

If the common denominator of two fractions is not immediately evident, you can always get a common denominator by multiplying the two denominators together.

Add. Check the work by going over it:

	a	*b*	*c*	*d*
(2)	$4\frac{1}{2}$ $5\frac{2}{3}$	$8\frac{1}{5}$ $2\frac{1}{2}$	$6\frac{1}{3}$ $1\frac{1}{4}$	$4\frac{1}{2}$ $4\frac{1}{3}$
(3)	$3\frac{1}{5}$ $7\frac{2}{3}$	$2\frac{2}{3}$ $5\frac{3}{4}$	$4\frac{1}{3}$ $5\frac{3}{4}$	$3\frac{1}{4}$ $3\frac{2}{3}$
(4)	$5\frac{4}{5}$ $1\frac{1}{2}$	$6\frac{1}{3}$ $7\frac{3}{5}$	$2\frac{4}{5}$ $4\frac{2}{3}$	$6\frac{2}{5}$ $2\frac{1}{4}$
(5)	$1\frac{2}{3}$ $1\frac{1}{4}$	$8\frac{3}{5}$ $5\frac{1}{2}$	$7\frac{1}{3}$ $2\frac{1}{2}$	$3\frac{1}{5}$ $9\frac{1}{2}$

For more practice, see rows 4 and 5 on page 154.

LEAST COMMON DENOMINATOR

To add three or more fractions like those at the right, you must first find the *least common denominator*, which is the *smallest* number that will contain 6, 3, and 8 exactly. This is done as shown below.

$\frac{1}{6} = \frac{4}{24}$
$\frac{1}{3} = \frac{8}{24}$
$\frac{3}{8} = \frac{9}{24}$
$\frac{21}{24} = \frac{7}{8}$

Try the largest denominator, 8, and see if it contains both 6 and 3 exactly; you see that 8 does not contain these numbers. Next multiply 8 by 2, which gives 16, and see if 16 contains both 6 and 3; 16 does *not* contain them. Then multiply 8 by 3, which gives 24, and see if 24 contains both 6 and 3; 24 *does* contain both 6 and 3, hence 24 is the *least common denominator*. Therefore change $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{3}{8}$ each to 24ths, as shown at the right, and add the fractions.

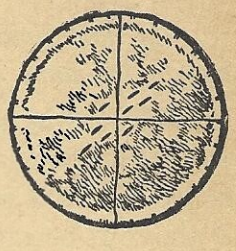
To find the least common denominator, see if the largest denominator contains the other denominators. If not, multiply it in turn by 2, 3, 4, etc. until you get a number that exactly contains each denominator.

Add. Check the work by going over it:

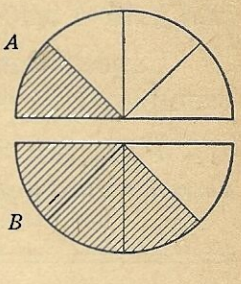
	a	*b*	*c*	*d*	*e*
(1)	$\frac{3}{4}$	$\frac{5}{8}$	$3\frac{3}{4}$	$6\frac{2}{3}$	$1\frac{3}{4}$
	$\frac{5}{6}$	$\frac{7}{12}$	$2\frac{2}{5}$	$2\frac{7}{10}$	$5\frac{2}{3}$
	$\frac{1}{2}$	$\frac{2}{3}$	$5\frac{1}{2}$	$2\frac{1}{6}$	$5\frac{1}{6}$
(2)	$\frac{7}{8}$	$\frac{3}{10}$	$1\frac{5}{6}$	$9\frac{1}{12}$	$2\frac{1}{6}$
	$\frac{3}{4}$	$\frac{1}{4}$	$4\frac{3}{8}$	$1\frac{2}{3}$	$7\frac{7}{8}$
	$\frac{1}{6}$	$\frac{4}{5}$	$3\frac{1}{2}$	$3\frac{3}{8}$	$4\frac{1}{3}$
(3)	$\frac{1}{8}$	$\frac{2}{3}$	$2\frac{1}{8}$	$4\frac{1}{10}$	$2\frac{1}{6}$
	$\frac{1}{3}$	$\frac{3}{10}$	$6\frac{5}{6}$	$3\frac{2}{3}$	$1\frac{2}{3}$
	$\frac{1}{2}$	$\frac{1}{5}$	$1\frac{1}{3}$	$8\frac{1}{2}$	$4\frac{1}{4}$
(4)	$\frac{1}{2}$	$\frac{5}{12}$	$2\frac{1}{2}$	$5\frac{1}{4}$	$6\frac{3}{8}$
	$\frac{1}{6}$	$\frac{3}{8}$	$4\frac{7}{8}$	$8\frac{4}{5}$	$4\frac{1}{2}$
	$\frac{1}{4}$	$\frac{1}{6}$	$2\frac{2}{3}$	$3\frac{1}{10}$	$1\frac{5}{6}$

For more practice, see row 7 on page 154.

(1) Do Improvement Test D3 on Subtraction. See page 142.



Fred cut a pie into halves; then he cut each half into 2 equal pieces, as shown at the right. Each piece is $\frac{1}{4}$ of the whole pie. Hence $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$. Another way to write $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{2} \times \frac{1}{2}$; this is read "$\frac{1}{2}$ times $\frac{1}{2}$" but it means $\frac{1}{2}$ of $\frac{1}{2}$. The sign $\times$ means "of" when it is written after a fraction.



The figure at the right shows 2 halves of a pie. Each half of the pie is cut into 4 equal parts; you see that each of these parts is $\frac{1}{8}$ of the whole pie.

Look at the half marked *A*. The shaded part is $\frac{1}{8}$ of the whole pie; the shaded part is also $\frac{1}{4}$ of $\frac{1}{2}$ of the pie. Hence $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$. Writing this another way, you get $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

Look at the half marked *B*. The shaded part is $\frac{3}{8}$ of the whole pie; it is also $\frac{3}{4}$ of $\frac{1}{2}$ of the pie. Hence $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$, or $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

In the work above, you have multiplied fractions but you have found the answers by the aid of drawings. A shorter way to find the same answers is as follows:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4} \qquad \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8} \qquad \frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

In each example, you first multiplied the numerators together and then multiplied the denominators together.

To multiply fractions, multiply the numerators together for a new numerator and the denominators together for a new denominator.

Multiply. Check by going over the work:

	a	*b*	*c*	*d*
(2)	$\frac{1}{3} \times \frac{1}{3} =$	$\frac{1}{2} \times \frac{1}{5} =$	$\frac{1}{8} \times \frac{3}{8} =$	$\frac{2}{3} \times \frac{2}{3} =$
(3)	$\frac{3}{4} \times \frac{1}{2} =$	$\frac{3}{4} \times \frac{5}{8} =$	$\frac{1}{6} \times \frac{1}{6} =$	$\frac{3}{5} \times \frac{3}{4} =$
(4)	$\frac{1}{4} \times \frac{1}{3} =$	$\frac{1}{2} \times \frac{3}{5} =$	$\frac{5}{6} \times \frac{1}{2} =$	$\frac{1}{4} \times \frac{3}{4} =$
(5)	$\frac{4}{5} \times \frac{2}{3} =$	$\frac{1}{8} \times \frac{1}{2} =$	$\frac{2}{3} \times \frac{1}{3} =$	$\frac{1}{3} \times \frac{1}{5} =$
(6)	$\frac{1}{2} \times \frac{1}{3} =$	$\frac{1}{4} \times \frac{1}{4} =$	$\frac{2}{5} \times \frac{2}{3} =$	$\frac{3}{4} \times \frac{3}{4} =$

CANCELLATION

You have learned this important principle:

If you divide both terms of a fraction by the same number, you get another fraction having the same value.

Thus $\frac{6}{8}$ is reduced to $\frac{3}{4}$ by dividing both 6 and 8 by 2. Another way to indicate this reduction is to *cancel* 2 from both 6 and 8 as shown at the right. To *cancel* 2 means to *divide by* 2. When you divide 6 by 2, cross out 6 and write 3 above it because $6 \div 2 = 3$. Similarly, when you divide 8 by 2, cross out 8 and write 4 below it.

$$\frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{3}{4}$$

Cancellation may also be used when fractions are multiplied. In the example at the right, divide 3 and 9 each by 3; then divide 4 and 8 each by 4.

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

When you cancel in multiplying fractions, you divide a numerator and a denominator by the same number.

Multiply, canceling when possible. Check the work by going over it:

	a	*b*	*c*	*d*
(1)	$\frac{2}{3} \times \frac{9}{16} =$	$\frac{5}{8} \times \frac{3}{10} =$	$\frac{7}{8} \times \frac{4}{5} =$	$\frac{1}{3} \times \frac{3}{8} =$
(2)	$\frac{15}{16} \times \frac{1}{3} =$	$\frac{4}{5} \times \frac{5}{8} =$	$\frac{5}{6} \times \frac{3}{10} =$	$\frac{3}{4} \times \frac{2}{5} =$
(3)	$\frac{3}{4} \times \frac{5}{6} =$	$\frac{3}{5} \times \frac{1}{3} =$	$\frac{2}{3} \times \frac{5}{8} =$	$\frac{5}{6} \times \frac{9}{10} =$
(4)	$\frac{3}{5} \times \frac{15}{16} =$	$\frac{5}{6} \times \frac{7}{10} =$	$\frac{3}{8} \times \frac{4}{5} =$	$\frac{2}{3} \times \frac{9}{10} =$
(5)	$\frac{5}{12} \times \frac{9}{10} =$	$\frac{8}{9} \times \frac{15}{16} =$	$\frac{3}{5} \times \frac{5}{6} =$	$\frac{4}{3} \times \frac{9}{16} =$
(6)	$\frac{6}{5} \times \frac{5}{12} =$	$\frac{1}{5} \times \frac{5}{16} =$	$\frac{1}{6} \times \frac{3}{8} =$	$\frac{2}{3} \times \frac{3}{16} =$
(7)	$\frac{3}{4} \times \frac{7}{12} =$	$\frac{2}{3} \times \frac{3}{8} =$	$\frac{1}{2} \times \frac{4}{5} =$	$\frac{5}{8} \times \frac{9}{10} =$
(8)	$\frac{4}{5} \times \frac{15}{16} =$	$\frac{7}{8} \times \frac{2}{3} =$	$\frac{5}{12} \times \frac{3}{10} =$	$\frac{5}{6} \times \frac{9}{16} =$

For more practice, see rows 2 and 3 on page 156.

Improvement Test — Multiplication H1

Time 4 min.

	a	*b*	*c*
(1)	312 908	463 287	791 652
(2)	584 436	849 193	627 507

-------- RIGHT OUT OF 6 SCORE -------- LINE E

Addition. Add upward. Check the work by adding downward:

	a	*b*	*c*	*d*	*e*	*f*
(3)	1299 4343 3724 7661 9329 2235 8584 4630 6409 2831 1672 4618	9728 2372 5900 4216 4373 1719 5895 9058 1610 4006 8570 3727	7141 8795 7880 9006 2369 5708 7776 4526 6290 1614 3924 2467	5561 2904 9195 8759 1240 8221 5439 3769 2833 1400 9263 2668	4569 3335 4360 8022 3888 6590 7322 2004 8313 1859 9475 9325	4043 7581 5702 1114 3458 1605 8957 2044 7549 7780 2531 1105

------ RIGHT OUT OF 6 SCORE -------- LINE E

For more practice, see page 151.

MULTIPLYING FRACTIONS

Any fraction may be regarded as a way of expressing the division of two numbers. Thus, $\frac{1}{2}$ means $1 \div 2$ and $\frac{3}{4}$ means $3 \div 4$. Similarly, $\frac{5}{1}$ means $5 \div 1$ or 5. Thus you see that a whole number like 5 may be written in the form of a fraction with a denominator of 1; that is, 5 may be written as $\frac{5}{1}$. In examples involving the multiplication of a fraction by a whole number or *vice versa*, it is often convenient to write the whole number in the form of a fraction, as shown above.

$$5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

Multiply. Reduce results when possible. Check by going over the work:

	a	*b*	*c*	*d*
(1)	$4 \times \frac{2}{3} =$	$7 \times \frac{3}{4} =$	$5 \times \frac{5}{6} =$	$9 \times \frac{2}{5} =$
(2)	$\frac{3}{8} \times 3 =$	$\frac{2}{3} \times 5 =$	$\frac{3}{4} \times 8 =$	$4 \times \frac{5}{12} =$
(3)	$\frac{2}{3} \times 10 =$	$\frac{7}{12} \times 36 =$	$\frac{3}{10} \times 9 =$	$\frac{5}{6} \times 9 =$
(4)	$\frac{3}{10} \times 25 =$	$10 \times \frac{5}{8} =$	$\frac{2}{3} \times 18 =$	$6 \times \frac{3}{4} =$
(5)	$6 \times \frac{7}{8} =$	$15 \times \frac{5}{6} =$	$4 \times \frac{11}{12} =$	$8 \times \frac{7}{10} =$
(6)	$\frac{3}{4} \times 10 =$	$\frac{3}{5} \times 12 =$	$8 \times \frac{2}{3} =$	$9 \times \frac{2}{3} =$
(7)	$\frac{3}{10} \times 16 =$	$32 \times \frac{5}{16} =$	$9 \times \frac{3}{8} =$	$8 \times \frac{7}{12} =$
(8)	$\frac{5}{12} \times 20 =$	$24 \times \frac{4}{5} =$	$6 \times \frac{7}{8} =$	$5 \times \frac{5}{8} =$
(9)	$\frac{9}{10} \times 16 =$	$14 \times \frac{7}{10} =$	$4 \times \frac{2}{5} =$	$\frac{3}{8} \times 12 =$
(10)	$\frac{9}{16} \times 48 =$	$16 \times \frac{7}{32} =$	$8 \times \frac{3}{4} =$	$\frac{5}{6} \times 10 =$

For more practice, see rows 4 and 5 on page 156.

(1) Do Improvement Test H2 on Multiplication. See page 144.

(2) Find the cost of $2\frac{3}{4}$ lb. of candy at 60¢ a pound.

First multiply 60 by 2 which gives 120. Next multiply 60 by $\frac{3}{4}$ which gives 45; write 45 under 120 so that the 5 comes under the 0 of 120. Then add 120 and 45. The candy costs 165¢ or $1.65.

A	B
60	$4\frac{1}{2}$
$2\frac{3}{4}$	3
120	12
45	$1\frac{1}{2}$
165	$13\frac{1}{2}$

(3) Tom is making a bookcase. Each shelf is $4\frac{1}{2}$ ft. long. How long a board will it take to make 3 shelves?

First multiply 4 by 3 which gives 12, as shown in *B*. Next multiply $\frac{1}{2}$ by 3 which gives $1\frac{1}{2}$. Then add 12 and $1\frac{1}{2}$ which gives $13\frac{1}{2}$. A board $13\frac{1}{2}$ ft. long is needed to make 3 shelves.

Multiply. Check the work by going over it:

	a	*b*	*c*	*d*	*e*
(4)	16 $2\frac{1}{5}$	20 $2\frac{1}{3}$	12 $3\frac{1}{3}$	10 $4\frac{3}{5}$	$3\frac{1}{2}$ 9
(5)	12 $3\frac{3}{5}$	15 $6\frac{2}{3}$	$18\frac{3}{4}$ 5	$11\frac{2}{3}$ 7	$4\frac{1}{8}$ 6
(6)	25 $4\frac{1}{6}$	$11\frac{3}{4}$ 7	10 $2\frac{3}{8}$	15 $3\frac{5}{8}$	8 $2\frac{3}{4}$
(7)	$10\frac{2}{3}$ 8	$14\frac{1}{8}$ 7	$2\frac{3}{4}$ 12	24 $2\frac{1}{5}$	7 $3\frac{2}{3}$
(8)	$16\frac{3}{4}$ 6	$10\frac{9}{10}$ 5	18 $4\frac{5}{8}$	$10\frac{1}{6}$ 8	$9\frac{1}{3}$ 5

For more practice, see rows 6 and 7 on page 156.

MULTIPLYING MIXED NUMBERS

Multiply, canceling when possible. Check by going over the work. Change mixed numbers to improper fractions as shown in ex. 1 a:

	a	*b*
(1)	$\frac{3}{4} \times 3\frac{1}{5} = \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{5} = \frac{12}{5} = 2\frac{2}{5}$	$1\frac{1}{4} \times 3\frac{1}{3} =$
(2)	$\frac{2}{3} \times 1\frac{3}{5} =$	$4\frac{1}{6} \times 1\frac{4}{5} =$
(3)	$\frac{3}{5} \times 8\frac{3}{4} =$	$1\frac{1}{5} \times 1\frac{1}{4} =$
(4)	$\frac{2}{3} \times 1\frac{3}{4} =$	$3\frac{1}{8} \times 2\frac{2}{5} =$

Copy on a sheet of paper, multiply, and check:

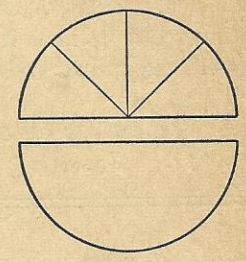
	a	*b*	*c*	*d*	*e*
(5)	$\frac{4}{5} \times 3\frac{1}{8}$	$\frac{2}{3} \times 2\frac{3}{4}$	$\frac{3}{8} \times 3\frac{1}{3}$	$4\frac{1}{5} \times \frac{5}{6}$	$2\frac{2}{3} \times 3\frac{3}{4}$
(6)	$\frac{1}{3} \times 1\frac{1}{2}$	$\frac{1}{6} \times 1\frac{1}{5}$	$\frac{3}{5} \times 4\frac{1}{6}$	$8\frac{1}{3} \times \frac{3}{10}$	$3\frac{1}{5} \times 3\frac{3}{4}$
(7)	$\frac{1}{8} \times 2\frac{2}{5}$	$\frac{2}{3} \times 1\frac{1}{2}$	$\frac{1}{4} \times 1\frac{1}{3}$	$1\frac{2}{3} \times \frac{3}{10}$	$6\frac{1}{4} \times 2\frac{4}{5}$
(8)	$\frac{1}{6} \times 4\frac{4}{5}$	$\frac{2}{5} \times 1\frac{7}{8}$	$\frac{2}{3} \times 4\frac{1}{5}$	$2\frac{2}{3} \times \frac{3}{4}$	$1\frac{7}{8} \times 1\frac{1}{5}$
(9)	$\frac{2}{3} \times 2\frac{5}{8}$	$\frac{4}{5} \times 7\frac{1}{2}$	$\frac{3}{8} \times 5\frac{1}{3}$	$1\frac{1}{5} \times \frac{5}{12}$	$1\frac{5}{6} \times 2\frac{2}{5}$
(10)	$\frac{3}{8} \times 2\frac{2}{3}$	$\frac{5}{6} \times 3\frac{1}{3}$	$\frac{1}{6} \times 7\frac{1}{5}$	$1\frac{1}{4} \times \frac{4}{5}$	$5\frac{1}{3} \times 1\frac{4}{5}$
(11)	$\frac{5}{16} \times 1\frac{1}{5}$	$\frac{2}{3} \times 1\frac{1}{8}$	$\frac{2}{5} \times 2\frac{1}{2}$	$2\frac{2}{3} \times \frac{3}{16}$	$1\frac{1}{5} \times 2\frac{1}{12}$
(12)	$\frac{2}{3} \times 3\frac{3}{4}$	$\frac{1}{4} \times 6\frac{2}{5}$	$\frac{5}{8} \times 1\frac{1}{5}$	$1\frac{7}{8} \times \frac{4}{5}$	$2\frac{2}{3} \times 2\frac{1}{16}$
(13)	$\frac{3}{4} \times 1\frac{1}{3}$	$\frac{3}{5} \times 3\frac{1}{3}$	$\frac{2}{3} \times 2\frac{1}{4}$	$1\frac{3}{5} \times \frac{5}{16}$	$5\frac{1}{4} \times 1\frac{1}{5}$

For more practice, see rows 8 to 10 on page 156.

(1) Do Improvement Test H3 on Multiplication. See page 144.

Inverting a Fraction. In the division of fractions, it is often necessary *to invert* a fraction, that is, to interchange the numerator and the denominator. If the fraction $\frac{2}{3}$ is inverted, it becomes $\frac{3}{2}$. If $\frac{1}{3}$ is inverted, it becomes $\frac{3}{1}$ which equals 3. If 5 is to be inverted, first write 5 as $\frac{5}{1}$, then invert $\frac{5}{1}$ which becomes $\frac{1}{5}$.

(2) Aunt Mary divides $\frac{1}{2}$ of a pie equally among 4 girls. What part of the whole pie does each girl get?



The figure at the right shows two halves of a pie with the upper half divided into 4 equal parts. Each girl gets one of these parts, each part being $\frac{1}{4}$ of $\frac{1}{2}$ of the pie. Show that each of these parts is also $\frac{1}{8}$ of the whole pie. Thus, by studying the figure, you find that each girl gets $\frac{1}{8}$ of the whole pie. Hence $\frac{1}{2} \div 4 = \frac{1}{8}$.

A shorter way to find that $\frac{1}{2} \div 4 = \frac{1}{8}$, without drawing a figure, is as follows: Invert the divisor 4, which gives $\frac{1}{4}$; then multiply $\frac{1}{2}$ by $\frac{1}{4}$ like this: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$. The answer, $\frac{1}{8}$, is the same as the one obtained above by drawing a figure. Thus you see that you can divide $\frac{1}{2}$ by 4 by *inverting* the divisor and *multiplying*. The work should be arranged as shown above at the right. Check the work by going over it.

$$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

(3) Mr. Thompson divides $1\frac{1}{2}$ melons equally between 2 boys. What part of a melon does each boy get?

If a mixed number is to be divided by a whole number, first change the mixed number to an improper fraction, as shown at the right. Then invert the divisor and multiply. Each boy gets $\frac{3}{4}$ of a melon.

$$1\frac{1}{2} \div 2 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

To divide a fraction or a mixed number by a whole number, invert the whole number and then multiply.

Copy on a sheet of paper, divide, and check:

	a	*b*	*c*	*d*	*e*
(4)	$\frac{1}{3} \div 2$	$\frac{7}{12} \div 2$	$\frac{5}{8} \div 10$	$2\frac{2}{5} \div 8$	$8\frac{1}{3} \div 15$
(5)	$\frac{5}{6} \div 3$	$\frac{3}{10} \div 6$	$\frac{7}{8} \div 21$	$3\frac{3}{4} \div 6$	$6\frac{3}{4} \div 18$
(6)	$\frac{3}{4} \div 3$	$\frac{15}{16} \div 5$	$\frac{5}{6} \div 15$	$2\frac{5}{8} \div 9$	$6\frac{2}{3} \div 10$
(7)	$\frac{5}{8} \div 9$	$\frac{9}{16} \div 6$	$\frac{5}{12} \div 10$	$2\frac{1}{2} \div 3$	$1\frac{1}{5} \div 12$
(8)	$\frac{2}{3} \div 8$	$\frac{9}{10} \div 2$	$\frac{9}{16} \div 18$	$7\frac{1}{2} \div 5$	$3\frac{3}{8} \div 15$
(9)	$\frac{5}{8} \div 2$	$\frac{5}{16} \div 4$	$\frac{3}{10} \div 12$	$7\frac{1}{5} \div 4$	$2\frac{2}{3} \div 24$

For more practice, see rows 11 and 12 on page 156.

DIVIDING BY A FRACTION

Mary has a piece of ribbon $3\frac{3}{4}$ yd. long. She wishes to cut it into pieces each $\frac{3}{4}$ yd. long. How many pieces will she get?

To find out, Mary has to divide $3\frac{3}{4}$ yd. by $\frac{3}{4}$ yd. but she does not know how to divide by a fraction like $\frac{3}{4}$. So she works the problem another way, like this: 1 yd. = 36 in., so $3\frac{3}{4}$ yd. = $3\frac{3}{4} \times 36$ in. or 135 in. $\frac{3}{4}$ yd. = $\frac{3}{4} \times 36$ in. or 27 in. Hence $3\frac{3}{4}$ yd. $\div \frac{3}{4}$ yd. is the same as 135 in. $\div$ 27 in., which equals 5. Therefore Mary will get 5 pieces of ribbon, each $\frac{3}{4}$ yd. long. Mary's solution gives the right answer but it takes too much time. A short way to work this problem is to invert the divisor and multiply just as you did on page 45. The short solution is shown above.

$$3\frac{3}{4} \div \frac{3}{4} = \frac{\overset{5}{\cancel{15}}}{\cancel{4}_{1}} \times \frac{\overset{1}{\cancel{4}}}{\cancel{3}_{1}} = 5$$

To divide by a fraction, invert the fraction and multiply.

Copy on a sheet of paper, divide, and check:

	a	*b*	*c*	*d*	*e*
(1)	$2\frac{1}{4} \div \frac{1}{8}$	$\frac{5}{8} \div \frac{5}{16}$	$\frac{3}{4} \div \frac{1}{12}$	$2\frac{1}{3} \div \frac{7}{8}$	$3\frac{3}{8} \div \frac{1}{6}$
(2)	$3\frac{3}{8} \div \frac{3}{4}$	$\frac{3}{16} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{9}{10}$	$6\frac{3}{4} \div \frac{3}{8}$	$10 \div \frac{5}{6}$
(3)	$3\frac{3}{4} \div \frac{3}{8}$	$\frac{2}{3} \div \frac{1}{6}$	$\frac{1}{8} \div \frac{5}{6}$	$2\frac{1}{4} \div \frac{9}{10}$	$8\frac{1}{3} \div \frac{1}{6}$
(4)	$4\frac{4}{5} \div \frac{4}{5}$	$\frac{3}{4} \div \frac{15}{16}$	$\frac{3}{8} \div \frac{1}{16}$	$6\frac{2}{3} \div \frac{15}{16}$	$1\frac{11}{16} \div \frac{3}{8}$
(5)	$5\frac{2}{5} \div \frac{1}{10}$	$\frac{5}{8} \div \frac{1}{4}$	$\frac{1}{4} \div \frac{5}{8}$	$7\frac{1}{5} \div \frac{3}{10}$	$42 \div \frac{7}{8}$
(6)	$1\frac{1}{8} \div \frac{3}{10}$	$\frac{2}{3} \div \frac{5}{6}$	$\frac{3}{5} \div \frac{5}{6}$	$2\frac{1}{2} \div \frac{2}{5}$	$6\frac{1}{2} \div \frac{1}{8}$
(7)	$4\frac{1}{6} \div \frac{5}{8}$	$\frac{1}{3} \div \frac{7}{12}$	$\frac{7}{16} \div \frac{3}{4}$	$1\frac{7}{8} \div \frac{3}{4}$	$1\frac{1}{2} \div \frac{3}{4}$
(8)	$6\frac{1}{2} \div \frac{1}{4}$	$\frac{1}{8} \div \frac{3}{8}$	$\frac{5}{12} \div \frac{1}{3}$	$1\frac{2}{3} \div \frac{15}{16}$	$1\frac{1}{6} \div \frac{7}{12}$
(9)	$2\frac{1}{16} \div \frac{11}{12}$	$\frac{7}{10} \div \frac{1}{3}$	$\frac{5}{8} \div \frac{1}{4}$	$1\frac{2}{5} \div \frac{4}{5}$	$15 \div \frac{3}{5}$
(10)	$4\frac{1}{5} \div \frac{9}{10}$	$\frac{5}{6} \div \frac{7}{12}$	$\frac{3}{8} \div \frac{15}{16}$	$3\frac{1}{3} \div \frac{1}{6}$	$27 \div \frac{9}{10}$
(11)	$5\frac{1}{3} \div \frac{5}{6}$	$\frac{7}{10} \div \frac{2}{5}$	$\frac{9}{10} \div \frac{3}{5}$	$6\frac{1}{4} \div \frac{5}{6}$	$2\frac{2}{3} \div \frac{1}{10}$
(12)	$4\frac{1}{5} \div \frac{3}{5}$	$\frac{15}{16} \div \frac{5}{12}$	$\frac{1}{3} \div \frac{1}{12}$	$1\frac{4}{5} \div \frac{7}{10}$	$6\frac{1}{8} \div \frac{7}{12}$
(13)	$2\frac{5}{8} \div \frac{7}{12}$	$\frac{7}{12} \div \frac{1}{8}$	$\frac{5}{6} \div \frac{5}{8}$	$4\frac{1}{8} \div \frac{3}{4}$	$9\frac{1}{3} \div \frac{4}{5}$

For more practice, see rows 13 to 15 on page 156.

IMPROVING BY PRACTICE

Improvement Test — Division S1

Time 4 min.

If any example has a remainder, write it as a fraction in the quotient:

	a	*b*	*c*
(1)	$48\overline{)20033}$	$69\overline{)36294}$	$34\overline{)14790}$
(2)	$51\overline{)35547}$	$82\overline{)43307}$	$63\overline{)40257}$

-------- RIGHT OUT OF 6

SCORE -------- LINE E

MIXED PRACTICE

Add	Subtract	Divide
(3) $7\frac{3}{4}$ + $2\frac{5}{6}$	**(4)** $11\frac{1}{4}$ − $8\frac{9}{16}$	**(5)** $8\overline{)9378}$

Multiply the following:

(6) $\frac{1}{8} \times \frac{1}{2} =$ $\quad \frac{4}{5} \times \frac{15}{16} =$ $\quad 12 \times \frac{3}{8} =$ $\quad \frac{2}{3} \times 20 =$

-------- RIGHT OUT OF 7

SCORE -------- LINE F

DIVIDING MIXED NUMBERS

(1) Do Improvement Test S2 on Division. See page 144.

Divide. Change the divisor to an improper fraction and invert, all in one step, as shown in ex. 2a:

	a	*b*
(2)	$2\frac{1}{4} \div 1\frac{3}{4} = \frac{9}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{7} = \frac{9}{7} = 1\frac{2}{7}$	$3\frac{3}{4} \div 5\frac{5}{8} =$
(3)	$1\frac{1}{3} \div 1\frac{1}{2} =$	$2\frac{2}{5} \div 3\frac{3}{4} =$
(4)	$5\frac{1}{3} \div 3\frac{1}{7} =$	$1\frac{5}{6} \div 1\frac{7}{8} =$
(5)	$2\frac{1}{2} \div 4\frac{1}{6} =$	$1\frac{2}{5} \div 2\frac{1}{10} =$

Copy, divide, and check:

	a	*b*	*c*	*d*	*e*
(6)	$2\frac{2}{3} \div 2\frac{4}{5}$	$16 \div 3\frac{1}{5}$	$\frac{5}{6} \div 2\frac{1}{3}$	$2\frac{3}{16} \div 1\frac{3}{4}$	$2\frac{2}{5} \div 3\frac{3}{5}$
(7)	$5\frac{1}{3} \div 1\frac{1}{6}$	$12 \div 3\frac{3}{4}$	$\frac{7}{8} \div 5\frac{1}{4}$	$6\frac{3}{4} \div 4\frac{1}{2}$	$6\frac{2}{3} \div 3\frac{1}{3}$
(8)	$2\frac{1}{6} \div 1\frac{5}{6}$	$5 \div 3\frac{1}{3}$	$\frac{2}{3} \div 4\frac{2}{3}$	$4\frac{2}{5} \div 3\frac{3}{10}$	$8\frac{1}{3} \div 3\frac{3}{4}$
(9)	$2\frac{1}{4} \div 3\frac{1}{3}$	$8 \div 3\frac{3}{5}$	$\frac{1}{5} \div 2\frac{1}{5}$	$3\frac{3}{4} \div 1\frac{9}{16}$	$6\frac{2}{5} \div 4\frac{4}{5}$
(10)	$6\frac{3}{4} \div 1\frac{4}{5}$	$42 \div 1\frac{3}{4}$	$\frac{9}{10} \div 7\frac{1}{2}$	$3\frac{3}{5} \div 2\frac{1}{4}$	$4\frac{7}{12} \div 3\frac{2}{3}$
(11)	$4\frac{1}{6} \div 1\frac{2}{3}$	$10 \div 8\frac{1}{3}$	$\frac{4}{5} \div 2\frac{1}{2}$	$1\frac{7}{8} \div 1\frac{1}{4}$	$4\frac{1}{6} \div 1\frac{1}{4}$
(12)	$2\frac{3}{5} \div 3\frac{9}{10}$	$24 \div 2\frac{2}{5}$	$\frac{5}{8} \div 6\frac{1}{4}$	$3\frac{1}{3} \div 2\frac{1}{12}$	$2\frac{2}{3} \div 1\frac{1}{5}$
(13)	$6\frac{1}{4} \div 2\frac{1}{2}$	$45 \div 4\frac{1}{2}$	$\frac{3}{4} \div 4\frac{1}{2}$	$7\frac{1}{3} \div 1\frac{5}{6}$	$2\frac{5}{8} \div 2\frac{1}{8}$
(14)	$18\frac{1}{3} \div 1\frac{5}{6}$	$25 \div 3\frac{3}{4}$	$\frac{7}{2} \div 3\frac{1}{2}$	$8\frac{3}{4} \div 3\frac{1}{2}$	$9\frac{1}{3} \div 2\frac{2}{3}$

For more practice, see rows 16 to 18 on page 156.

(1) Do Improvement Test S3 on Division. See page 144.

opy; then add or subtract. Check the work:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
(2)	$9\frac{3}{4}$	$2\frac{3}{8}$	$7\frac{7}{16}$	$8\frac{3}{4}$	$6\frac{1}{10}$	$8\frac{1}{2}$	$9\frac{3}{5}$	$5\frac{11}{16}$	$4\frac{7}{8}$
	$4\frac{1}{2}$	$1\frac{1}{4}$	$5\frac{5}{8}$	$4\frac{3}{4}$	$2\frac{2}{5}$	$1\frac{4}{5}$	$8\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{1}{16}$
(3)	$7\frac{1}{3}$	$8\frac{5}{12}$	$4\frac{1}{2}$	$9\frac{3}{4}$	$6\frac{1}{2}$	$5\frac{4}{5}$	14	$9\frac{1}{4}$	$7\frac{3}{4}$
	$3\frac{1}{2}$	$3\frac{1}{3}$	$1\frac{7}{8}$	$5\frac{1}{12}$	$4\frac{3}{10}$	$1\frac{3}{5}$	$3\frac{9}{10}$	$6\frac{3}{4}$	$2\frac{2}{3}$
(4)	$9\frac{7}{16}$	6	$8\frac{1}{2}$	$8\frac{7}{12}$	$5\frac{5}{6}$	$7\frac{1}{2}$	$3\frac{9}{10}$	$5\frac{3}{8}$	$4\frac{3}{4}$
	$3\frac{3}{16}$	$3\frac{5}{6}$	$5\frac{1}{3}$	$2\frac{1}{3}$	$2\frac{3}{8}$	$4\frac{1}{5}$	$1\frac{1}{4}$	$4\frac{7}{8}$	$3\frac{1}{6}$

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oy; then multiply or divide, as indicated. Check the work:

	a	*b*	*c*	*d*	*e*
5)	$\frac{3}{5} \times \frac{1}{2}$	$\frac{5}{6} \times \frac{3}{10}$	$\frac{7}{8} \times 10$	$24 \times 2\frac{1}{2}$	$3\frac{3}{8} \times 1\frac{1}{3}$
6)	$\frac{5}{8} \times \frac{2}{5}$	$\frac{7}{8} \times 1\frac{1}{2}$	$\frac{3}{5} \times 32$	$15 \times 1\frac{5}{6}$	$2\frac{2}{3} \times 2\frac{1}{16}$
7)	$\frac{2}{3} \times \frac{3}{10}$	$\frac{3}{5} \times 3\frac{1}{3}$	$\frac{1}{2} \times 2\frac{2}{5}$	$\frac{3}{10} \times 8\frac{1}{3}$	$6\frac{3}{4} \times 2\frac{2}{3}$
8)	$6 \times \frac{3}{5}$	$3 \times 3\frac{1}{4}$	$\frac{1}{5} \times 8\frac{3}{4}$	$\frac{7}{12} \times 48$	$1\frac{2}{3} \times 2\frac{1}{10}$
9)	$\frac{3}{8} \times \frac{2}{3}$	$\frac{2}{3} \times 4\frac{1}{2}$	$\frac{4}{5} \times 3\frac{3}{8}$	$\frac{2}{3} \times 17$	$6\frac{1}{4} \times 2\frac{2}{5}$
0)	$\frac{5}{8} \div 5$	$6 \div \frac{2}{3}$	$\frac{5}{8} \div 1\frac{7}{8}$	$3\frac{3}{4} \div \frac{1}{8}$	$2\frac{2}{5} \div 2\frac{1}{10}$
(11)	$\frac{2}{3} \div 7$	$8 \div \frac{4}{5}$	$9 \div 1\frac{1}{2}$	$2\frac{2}{5} \div 8$	$5\frac{5}{6} \div 1\frac{2}{3}$
(12)	$\frac{5}{8} \div \frac{15}{16}$	$4 \div \frac{1}{4}$	$\frac{3}{4} \div 1\frac{1}{5}$	$4\frac{4}{5} \div \frac{3}{10}$	$4\frac{1}{2} \div 3\frac{3}{4}$
(13)	$\frac{3}{8} \div \frac{3}{4}$	$5 \div \frac{1}{2}$	$\frac{1}{6} \div 4\frac{1}{2}$	$5\frac{1}{2} \div \frac{1}{4}$	$6\frac{1}{2} \div 3\frac{1}{4}$
(14)	$\frac{1}{4} \div \frac{1}{8}$	$9 \div \frac{2}{3}$	$\frac{4}{5} \div 18$	$3\frac{1}{5} \div \frac{1}{10}$	$2\frac{1}{4} \div 1\frac{7}{8}$

For more practice, see pages 154 to 156.

DIAGNOSTIC TEST

Add the following:

	a	*b*	*c*	*d*
(1)	$2\frac{1}{8}$	$7\frac{5}{8}$	$1\frac{1}{2}$	$4\frac{1}{2}$
	$4\frac{3}{4}$	$2\frac{1}{6}$	$7\frac{3}{10}$	$6\frac{2}{3}$
	$5\frac{1}{2}$	$4\frac{1}{4}$	$4\frac{4}{5}$	$3\frac{3}{4}$

Subtract the following:

(2)	$9\frac{7}{8}$	$15\frac{1}{3}$	$8\frac{1}{5}$	$10\frac{9}{16}$
	$2\frac{1}{2}$	$5\frac{5}{6}$	$2\frac{3}{4}$	$5\frac{1}{4}$

Multiply the following:

	a	*b*	*c*
(3)	$\frac{2}{3} \times 15 =$	$8 \times 4\frac{2}{3} =$	$6\frac{2}{3} \times \frac{9}{16} =$
(4)	$\frac{7}{8} \times \frac{1}{2} =$	$\frac{5}{16} \times 3\frac{1}{5} =$	$4\frac{1}{5} \times 3\frac{1}{3} =$

Divide the following:

(5)	$\frac{5}{8} \div 10 =$	$6\frac{1}{4} \div 10 =$	$\frac{9}{10} \div \frac{3}{8} =$
(6)	$15 \div \frac{3}{4} =$	$4\frac{7}{8} \div 2\frac{1}{6} =$	$9\frac{3}{4} \div 4\frac{7}{8} =$

-------- RIGHT OUT OF 20

SCORE -------- LINE T

REMEDIAL EXERCISES

See page 138.

Ex. 1 *a*–1 *d*	p. 154, 1–7	Ex. 4 *a*	p. 156, 1–3	Ex. 5 *c*	p. 156, 13–15
Ex. 2 *a*–2 *d*	p. 155, 1–8	Ex. 4 *b*, *c*	p. 156, 8–10	Ex. 6 *a*	p. 156, 13–15
Ex. 3 *a*, *b*, *c*	p. 156, 4–10	Ex. 5 *a*, *b*	p. 156, 11–12	Ex. 6 *b*, *c*	p. 156, 16–18

Improvement Test — Addition B1 Time 4 min.

	a	*b*	*c*	*d*	*e*
(1)	1923	2240	5031	3367	3250
	5056	8476	3664	1415	5177
	7899	3802	2347	2152	1236
	6257	5768	2356	8379	8936
	2784	4175	8494	6926	7265
	4675	4593	1569	9207	3459
	8463	9989	9887	5181	8491
	2538	4614	2532	5867	1745

-------- RIGHT OUT OF 5 SCORE -------- LINE D

DECIMALS

A decimal is a short way of writing a fraction like $\frac{7}{10}$ or $\frac{19}{100}$ without writing the denominator. Thus .7 means $\frac{7}{10}$ and is read 7 *tenths*. .15 means $\frac{15}{100}$ and is read 15 *hundredths*. .165 means $\frac{165}{1000}$ and is read 165 *thousandths*. .0073 means $\frac{73}{10000}$ and is read 73 *ten-thousandths*. The period in front of .7, .15, etc. is called a *decimal point*. A zero is often placed before a decimal like .35 to call attention to the decimal point. Thus, .35 may be written as 0.35.

hundreds	tens	units	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
3	1	5	.	2	0	7	6	5	3

Each decimal place has a name, as shown at the right. To read a decimal, read it as a whole number and give it the name of its *last* decimal place. For example, .46 is read *forty-six hundredths* because the last figure, 6, is in the *hundredths'* place; likewise, .375 is read *three hundred seventy-five thousandths* because the last figure, 5, is in the *thousandths'* place. The number 5.413 is called a mixed decimal and is read *five and four hundred thirteen thousandths*. In science and industry 5.413 is often read *five-point-four-one-three*.

The value of each digit in a number depends upon its position in the number. For example, the 3 in 263 represents 3 *units;* the 3 in 236 represents 3 *tens* or 30, and the 3 in 326 represents 3 *hundreds* or 300. Each time that the 3 is moved one place to the left, its value becomes 10 times as great as before. Likewise, the 3 in .723 represents 3 *thousandths* or .003; the 3 in .732 represents 3 *hundredths* or .03; while the 3 in .372 represents 3 *tenths* or .3. In these numbers it is also evident that each time the 3 is moved one place to the left, its value becomes 10 times as great as before.

In everyday life many of our units of measurement are now divided decimally. For example, a weather report may state that .36 in. of rain fell, a camera may take a picture in .001 sec., and an automobile may travel 7.3 mi.

ADDING DECIMALS

A decimal like 2.35 represents a measurement of some kind such as 2.35 in., or 2.35 mi., or 2.35 lb. In \$2.35, the .35 represents $\frac{35}{100}$ of a dollar which means 35 cents. A *cent* is $\frac{1}{100}$ of a dollar.

Decimals are added like whole numbers and most of the practical problems in the addition of decimals are such that all the numbers in a given problem have the same number of decimal places, as shown in *A* at the right. An example like *B* is usually impractical when the numbers refer to measurements; for example, if the numbers in *B* represent feet, 3.175 ft. would be a very precise measurement while 4.1 ft. would be somewhat crude; in this case it would be impractical to add precise measurements to crude measurements and expect the result to be dependable. If all the measurements in *B* were made with equal care, to the nearest thousandth of a foot, the numbers should be written as shown in *C* to indicate such care.

A	B	C
3.276	3.175	3.175
4.135	2.28	2.280
1.427	4.1	4.100
8.838	9.555	9.555

Examples like *B*, where the addends have a "ragged" right-hand margin, may occur in using an interest table or in computing interest by the 6% method.

Add upward. Check by adding downward:

	a	*b*	*c*	*d*	*e*	*f*
(1)	\$8.12	\$4.65	\$9.99	\$6.62	\$1.95	\$8.37
	1.33	2.36	7.60	4.86	7.46	.87
	9.38	9.02	.15	3.27	1.06	5.25
	2.20	4.34	4.29	5.30	.20	.30
	.87	6.89	7.00	9.18	8.55	4.07
	7.07	.81	8.65	.92	2.38	4.56
	4.94	3.08	.90	7.54	7.82	1.53
	.10	9.20	7.04	1.06	.20	.20
	2.19	.47	.85	3.11	5.36	8.07
	.41	3.13	2.16	.29	6.39	1.19
(2)	1.250	2.75	1.167	3.375	1.667	1.5
	4.333	1.5	2.25	2.500	3.75	1.875
	3.500	.375	2.833	1.625	2.833	4.75
(3)	67.89	481.0	6.225	1.417	25.50	9.451
	48.16	520.3	4.398	7.969	83.26	4.840
	70.24	721.3	1.962	7.782	25.84	3.055

(1) Do Improvement Test B2 on Addition. See page 141.

Subtracting Decimals. Subtract. Check the work by adding upward:

	a	*b*	*c*	*d*	*e*	*f*
(2)	5.5 3.5	7.3 3.8	4.8 1 3.8 5	6.4 4 4.7 5	7.1 5 0 5.1 9 5	9.1 3 3 7.1 2 5
(3)	8.7 2.3	9.1 6.0	3.6 7 2.7 5	9.2 8 5.9 2	4.2 8 7 3.2 9 9	6.0 2 5 .3 7 5
(4)	4. 2.5	5. 3.8	6. 2.7 5	4. 2.8 4	4.8 7 5 2.	9.4 3 4 3.0 9 7

Changing and Comparing Decimals. When you change a fraction like $\frac{3}{4}$ to $\frac{6}{8}$, you change the *name* of the fraction from *fourths* to *eighths* but you do not change the value of the fraction because $\frac{3}{4} = \frac{6}{8}$.

Every decimal has the same name as its *understood* denominator. Thus .35 has the name *hundredths* while .467 has the name *thousandths*. You can change the name of a decimal without changing its value by annexing or removing zeros at the right of the decimal. For example, .35 can be changed to .350 by annexing a zero after .35; in this case the name is changed from *hundredths* to *thousandths* but the value has not been changed; .35 = .350 since $\frac{35}{100} = \frac{350}{1000}$.

To compare two decimals, they must have the same name. For example, to compare .35 and .215, first change .35 to .350; .215 and .350 now have the same name which is *thousandths*. If you then compare .350 and .215, you see that .350 is larger than .215.

In each exercise, change one of the decimals so that both decimals will have the same name. Then tell which decimal is larger by drawing a line under it:

	a	*b*	*c*	*d*
(5)	.38 or .5	.08 or .3	.369 or .52	.9275 or .930
(6)	.41 or .4	.57 or .6	.436 or .43	.3007 or .304
(7)	.18 or .2	.34 or .4	.060 or .06	.0582 or .060
(8)	.60 or .6	.07 or .1	.700 or .77	.0800 or .008
(9)	.76 or .8	.81 or .8	.524 or .53	.0132 or .003

CHANGING DECIMALS TO FRACTIONS

A decimal like .35 is a fraction whose denominator is *understood* but not written. A common fraction, like $\frac{7}{8}$, is one whose denominator is written. A decimal is easily changed to a common fraction by writing the understood denominator of the decimal. Thus .35 becomes $\frac{35}{100}$; if both terms of $\frac{35}{100}$ are divided by 5, this fraction reduces to $\frac{7}{20}$. Hence $.35 = \frac{35}{100} = \frac{7}{20}$. Decimals like $.37\frac{1}{2}$ and $.66\frac{2}{3}$ are changed to common fractions as follows:

$$.37\tfrac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8} \qquad .66\tfrac{2}{3} = \frac{66\frac{2}{3}}{100} = \frac{200}{300} = \frac{2}{3}$$

Change to common fractions. Reduce results to lowest terms:

	a	*b*	*c*	*d*	*e*
(1)	.4 =	.3 =	.78 =	.93 =	.375 =
(2)	.6 =	.1 =	.45 =	.27 =	.125 =
(3)	.2 =	.05 =	.28 =	.50 =	.875 =
(4)	.7 =	.11 =	.30 =	.49 =	.625 =
(5)	.12 =	.32 =	.25 =	.72 =	$.37\frac{1}{2}$ =
(6)	.08 =	.54 =	.85 =	.61 =	$.62\frac{1}{2}$ =
(7)	.03 =	.15 =	.38 =	.55 =	$.12\frac{1}{2}$ =
(8)	.17 =	.66 =	.18 =	.82 =	$.87\frac{1}{2}$ =
(9)	.21 =	.48 =	.65 =	.42 =	$.66\frac{2}{3}$ =
(10)	.35 =	.75 =	.37 =	.95 =	$.33\frac{1}{3}$ =
(11)	.57 =	.69 =	.44 =	.23 =	$.06\frac{1}{4}$ =

(1) Do Improvement Test B3 on Addition. See page 141.

When you multiply \$1.15 by 5, you are multiplying a decimal by a whole number. The decimal point in the product \$5.75 is placed before the last two figures to separate the dollars from the cents. You see that the product has the same number of decimal places as the multiplicand \$1.15.

$$\begin{array}{r} \$1.15 \\ \times\ \ 5 \\ \hline \$5.75 \end{array}$$

To multiply a decimal by a whole number, first multiply as with whole numbers. Then, beginning at the right, point off as many decimal places in the product as there are decimal places in the multiplicand.

Copy on a separate sheet of paper. Multiply and check the work:

	a	*b*	*c*	*d*	*e*
(2)	$5 \times .84$	$2 \times .019$	8×7.225	19×3.17	37×54.3
(3)	$8 \times .16$	$2 \times .324$	6×2.465	18×2.34	$39 \times .516$
(4)	4×7.9	5×3.15	4×6.375	37×2.09	$23 \times .073$
(5)	$3 \times .003$	$4 \times .125$	8×1.295	24×14.5	$15 \times .092$
(6)	$9 \times .079$	$9 \times .834$	7×3.141	$16 \times .095$	$35 \times .016$

Multiplying by 10 and 100. To multiply a number by 10, 100, or 1000, move the decimal point as many places to the right as there are zeros in the multiplier. Multiply the following mentally:

	a	*b*	*c*
(7)	$10 \times 3.75 =$	$100 \times 9.62 =$	$1000 \times .388 =$
(8)	$10 \times 28.4 =$	$100 \times 8.25 =$	$1000 \times 7.28 =$
(9)	$10 \times 48.7 =$	$100 \times 87.5 =$	$1000 \times 67.5 =$
(10)	$10 \times .625 =$	$100 \times .453 =$	$1000 \times 2.258 =$
(11)	$10 \times 326 =$	$100 \times 389 =$	$1000 \times .0376 =$
(12)	$10 \times 1.433 =$	$100 \times .0427 =$	$1000 \times .0006 =$
(13)	$10 \times .0327 =$	$100 \times .0036 =$	$1000 \times 14.93 =$

For more practice, see rows 1 to 5 on page 157.

MULTIPLYING BY A DECIMAL

In the summer Frederick works in a store and is paid \$1.20 an hour. How much does he earn in 3.5 hr.?

To find what Frederick earns, \$1.20 must be multiplied by 3.5. To do this, first multiply as with whole numbers, which gives 4200. Then locate the position of the decimal point in the answer, which may be done by a mental estimate as follows: Since Frederick makes a little over \$1 an hour, he must earn more than \$3 in 3.5 hr. but he cannot earn \$42 in that time. Hence the decimal point must be placed after the 4 of 4200 rather than after the 2. The result is \$4.200, or \$4.20.

\$1.20
3.5
600
360
\$4.200

The position of the decimal point in the answer could also be determined by using the following rule:

In multiplying decimals, begin at the right and point off as many decimal places in the product as there are decimal places in the multiplier and multiplicand together.

In the example above, there are 2 decimal places in \$1.20 and 1 place in 3.5, which makes 3 places all together. Hence 3 places should be pointed off in the product, beginning at the right.

In the example at the right, there are 3 decimal places in .045 and 2 places in .03, making 5 places in all. Hence 5 places must be pointed off in the product. To get 5 places in the product, write 2 zeros before 135, which gives .00135.

.045
.03
.00135

Copy on a separate sheet of paper and multiply. Check the work:

	a	*b*	*c*	*d*	*e*
(1)	.13 × 4.2	.18 × 394	.37 × 4.8	2.4 × 3.14	.19 × 9.84
(2)	.76 × 2.7	.63 × .082	.79 × .87	6.2 × .051	.53 × .048
(3)	.34 × .52	.75 × 5.9	.06 × .004	.84 × 29.7	1.7 × .127
(4)	4.2 × .63	.07 × 6.3	.13 × .075	.39 × .189	5.6 × .376
(5)	.98 × 8.7	1.4 × .52	5.9 × 5.7	.89 × 3.14	.08 × .09
(6)	7.1 × .56	.09 × 4.5	3.5 × 2.3	.003 × 7.98	8.2 × .076
(7)	.02 × .89	5.4 × 7.5	.69 × .96	7.8 × .201	4.7 × 63.1
(8)	.03 × .531	1.9 × .24	.06 × .147	.025 × 440	9.2 × 5.78
(9)	9.5 × 68	.67 × .43	5.5 × .169	.006 × 245	7.3 × .489
(10)	5.2 × 3.2	.02 × .215	.81 × .546	.001 × 7.98	2.5 × .024

For more practice, see rows 6 to 13 on page 157.

NOTE. Exercises in the multiplication and division of decimals may be checked, if desired, by casting out 9's. In using this check the decimal points are ignored. When applied to decimals, the check of 9's tells whether or not the computation is correct but it does not check the placing of the decimal point in the answer.

Improvement Test — Subtraction E1

Time 4 min.

	a	*b*	*c*	*d*
(1)	970085 421569	410868 332108	441260 158467	953110 434174
(2)	672380 266819	900265 352462	800000 325371	663526 346781
(3)	734548 360499	389476 300494	951237 494301	912862 630879

-------- RIGHT OUT OF 12

SCORE -------- LINE L

MIXED PRACTICE

Multiply

(4) $5.72 × $4\frac{3}{4}$

Multiply

(5) $\frac{3}{16} \times \frac{2}{3}$

Add

(6)
$2.56
8.31
1.44
5.00
2.36
1.08
4.50

Subtract

(7)
6.835
1.067

Divide

(8) $17\overline{)5850}$

Multiply

(9)
4.16
.03

-------- RIGHT OUT OF 6

SCORE -------- LINE E

DIVIDING DECIMALS

A baseball team bought some supplies for $24.30. This expense was divided equally among the 9 boys on the team. What was each boy's share?

When you divide $24.30 by 9, you are dividing a decimal. To do this, first divide 24 by 9, which gives 2 as the first figure of the quotient. *Be sure to write 2 over the 4 of 24*; that is, write the first quotient figure over the *last* figure of the part of the dividend that you used. The other quotient figures are easily obtained. The decimal point in the answer *must be placed directly over the decimal point* in 24.30. Each boy's share of the total expense is $2.70. Instead of writing out the steps as shown in *A*, the work may be done mentally, if preferred, as shown in *B*.

A

$$\begin{array}{r} 2.70 \\ 9\overline{)24.30} \\ 18 \\ \hline 63 \\ 63 \\ \hline 0 \\ 0 \\ \hline \end{array}$$

In dividing a decimal by a whole number, write the first quotient figure directly above the last figure of the first partial dividend. Put the decimal point in the quotient directly above the decimal point in the dividend.

B

$$\begin{array}{r} 2.70 \\ 9\overline{)24.30} \end{array}$$

Divide. Check by going over the work:

	a	*b*	*c*	*d*	*e*
(1)	$6\overline{)9.36}$	$7\overline{).917}$	$8\overline{)58.4}$	$6\overline{)3.66}$	$9\overline{)58.23}$
(2)	$5\overline{).835}$	$6\overline{)46.8}$	$4\overline{)8.16}$	$9\overline{)34.2}$	$7\overline{)6.503}$
(3)	$7\overline{)5.04}$	$8\overline{).424}$	$6\overline{)83.4}$	$8\overline{)4.32}$	$6\overline{)149.4}$
(4)	$8\overline{).928}$	$9\overline{)4.86}$	$9\overline{)92.7}$	$4\overline{)63.2}$	$7\overline{)15.89}$

For more practice, see rows 14 to 16 on page 157.

(1) Do Improvement Test E2 on Subtraction. See page 143.

Tom earned \$15 in 4 days. How much did he earn a day?

You could divide as shown in *A*, getting a quotient of $\$3\frac{3}{4}$; but if you annex zeros to \$15, writing it as \$15.00, you avoid the fraction, as shown in *B*. Tom earns \$3.75 a day.

A

$$4\overline{)\$15}\quad \$3\tfrac{3}{4}$$

B

$$4\overline{)\$15.00}\quad \$3.75$$

Divide, writing a decimal point and enough 0's after the dividend to make the quotient come out exact. Check by multiplying on a separate sheet of paper:

	a	*b*	*c*	*d*
(2)	$6\overline{)33}$	$5\overline{)64}$	$5\overline{)299}$	$4\overline{)342}$
(3)	$4\overline{)18}$	$8\overline{)44}$	$6\overline{)75}$	$5\overline{)153}$
(4)	$8\overline{)20}$	$75\overline{)180}$	$44\overline{)154}$	$95\overline{)437}$
(5)	$8\overline{)38}$	$64\overline{)304}$	$28\overline{)231}$	$92\overline{)299}$

For more practice, see rows 17 to 20 on page 157 and rows 1 to 5 on page 158.

ROUNDING OFF RESULTS

In example *A* the answer, 1.648, is carried to three decimal places to avoid a remainder. If you wish to express this result *correct to the nearest hundredth*, it is called 1.65, because 1.648 is closer to 1.65 than to 1.64. In this case, 1.648 is said to be rounded off to the *nearest hundredth*. In example *B*, the result, 1.732, becomes 1.73 when rounded off *to the nearest hundredth*, because 1.732 is closer to 1.73 than to 1.74.

A. $5\overline{)8.240}$ → 1.648

In the same way, in example *C*, the quotient, 1.49, has been carried to two decimal places. If 1.49 is rounded off *to the nearest tenth*, it becomes 1.5, because 1.49 is closer to 1.5 than to 1.4. Similarly, in example *D*, the result, 1.33, becomes 1.3 *to the nearest tenth*, since 1.33 is closer to 1.3 than to 1.4.

B. $5\overline{)8.660}$ → 1.732

C. $6\overline{)8.95}$ → 1.49

D. $3\overline{)4.00}$ → 1.33

Carry the quotient to three decimal places to get a result correct to the nearest hundredth, or to two decimal places for a result correct to the nearest tenth. Then round off as follows: If the last figure of the quotient is less than 5, drop it; if the last figure is 5 or more, drop it but make the figure before it 1 larger.

After each number write the number rounded off to the nearest hundredth:

	a	*b*	*c*	*d*	*e*
(1)	2.168	5.171	1.342	7.396	4.833
(2)	4.034	8.790	3.195	3.207	2.667
(3)	4.508	5.301	8.214	1.753	2.385
(4)	9.618	8.415	4.357	2.599	1.700
(5)	7.098	4.292	5.706	8.602	6.955
(6)	8.097	9.999	4.009	3.070	3.001

After each number write the number rounded off to the nearest tenth:

(7)	6.08	3.25	5.96	3.91	2.74
(8)	8.13	8.97	7.50	4.07	6.92
(9)	4.85	3.05	5.93	8.99	1.56
(10)	9.21	5.02	8.49	2.95	4.80
(11)	8.39	8.70	9.99	7.09	6.66

(1) Do Improvement Test E3 on Subtraction. See page 143.

Copy and divide. In ex. 2 to 9, carry the quotient to 2 decimal places; then round off the result to the nearest tenth. In ex. 10 to 15, carry the quotient to 3 decimal places, rounding off the result to the nearest hundredth:

	a	*b*	*c*	*d*	*e*
(2)	19 ÷ 3	295 ÷ 7	368 ÷ 84	28.43 ÷ 35	9124.5 ÷ 586
(3)	41 ÷ 6	197 ÷ 8	665 ÷ 79	337.4 ÷ 61	786.31 ÷ 217
(4)	47 ÷ 8	417 ÷ 4	176 ÷ 21	172.8 ÷ 43	388.16 ÷ 145
(5)	80 ÷ 6	176 ÷ 3	399 ÷ 37	68.43 ÷ 75	9271.3 ÷ 459
(6)	56 ÷ 3	131 ÷ 8	384 ÷ 43	35.72 ÷ 38	117.25 ÷ 374
(7)	98 ÷ 9	452 ÷ 7	436 ÷ 19	983.7 ÷ 92	4923.7 ÷ 861
(8)	38 ÷ 7	193 ÷ 9	132 ÷ 56	247.3 ÷ 59	712.33 ÷ 692
(9)	79 ÷ 9	147 ÷ 8	837 ÷ 48	113.3 ÷ 26	1324.6 ÷ 173
(10)	48 ÷ 7	265 ÷ 6	183 ÷ 32	242.7 ÷ 53	2434.1 ÷ 814
(11)	37 ÷ 8	845 ÷ 6	136 ÷ 23	124.6 ÷ 16	3692.8 ÷ 297
(12)	76 ÷ 6	874 ÷ 9	528 ÷ 39	61.57 ÷ 53	142.73 ÷ 351
(13)	43 ÷ 9	523 ÷ 7	712 ÷ 46	321.9 ÷ 64	982.76 ÷ 689
(14)	29 ÷ 3	626 ÷ 9	217 ÷ 55	98.21 ÷ 36	4418.8 ÷ 166
(15)	33 ÷ 7	131 ÷ 6	188 ÷ 72	785.3 ÷ 94	143.26 ÷ 547

DIVIDING BY 10, 100, AND 1000

To divide a number by 10, 100, or 1000, move the decimal point as many places to the left as there are zeros in the divisor. Divide the following mentally:

	a	*b*	*c*
(16)	45.1 ÷ 10 =	432 ÷ 100 =	2436 ÷ 1000 =
(17)	678 ÷ 10 =	27.8 ÷ 100 =	469.2 ÷ 1000 =
(18)	1.25 ÷ 10 =	8.51 ÷ 100 =	5729 ÷ 1000 =
(19)	491 ÷ 10 =	163.2 ÷ 100 =	80.53 ÷ 1000 =
(20)	.613 ÷ 10 =	28.49 ÷ 100 =	6.123 ÷ 1000 =
(21)	.058 ÷ 10 =	2.435 ÷ 100 =	22.589 ÷ 1000 =

For more practice, see rows 6 to 15 on page 158.

MIXED PRACTICE

Add

(1)
$$\begin{array}{r} \$17.38 \\ 21.09 \\ 15.37 \\ 48.00 \\ 21.64 \\ 10.75 \\ 68.34 \\ 11.07 \\ \underline{23.45} \end{array}$$

Divide

(2) $79\overline{)73786}$

Add

(3)
$$\begin{array}{r} \frac{1}{2} \\ \frac{2}{3} \\ \frac{5}{12} \\ \underline{\frac{1}{6}} \end{array}$$

Multiply

(4)
$$\begin{array}{r} 265.3 \\ \underline{1.85} \end{array}$$

Subtract

(5)
$$\begin{array}{r} 700000 \\ \underline{480009} \end{array}$$

Divide

(6) $4\overline{)26.852}$

Multiply

(7) $\frac{9}{10} \times \frac{5}{6} =$

Add

(8)
$$\begin{array}{r} 6\frac{3}{16} \\ \underline{4\frac{5}{8}} \end{array}$$

Divide the following:

	a	*b*	*c*
(9)	$8\frac{1}{3} \div 15 =$	$2\frac{1}{16} \div 2\frac{3}{4} =$	$\frac{9}{10} \div \frac{9}{16} =$

Add

(10)
$$\begin{array}{r} 2\frac{5}{6} \\ 1\frac{3}{4} \\ \underline{4\frac{1}{2}} \end{array}$$

Subtract

(11)
$$\begin{array}{r} 4\frac{3}{4} \\ \underline{2\frac{1}{3}} \end{array}$$

Multiply

(12)
$$\begin{array}{r} 20 \\ \underline{5\frac{3}{4}} \end{array}$$

Divide; round off results to the nearest tenth:

	a	*b*	*c*	*d*
(13)	$63\overline{)283}$	$49\overline{)306}$	$28\overline{)194}$	$72\overline{)284.9}$

-------- RIGHT OUT OF 18

SCORE -------- LINE R

Improvement Test — Multiplication J1 Time 4 min.

	a	*b*	*c*
(1)	2186 × 55	3014 × 78	5967 × 64
(2)	7852 × 37	6879 × 92	2918 × 49

-------- RIGHT OUT OF 6 SCORE -------- LINE E

CHANGING FRACTIONS TO DECIMALS

Today, in shops and factories, machinists have delicate instruments with which they make measurements that are correct within 100ths, 1000ths, or 10,000ths of an inch. In other words, in modern shops, machinists measure in terms of *decimal parts* of an inch. On the other hand, the draftsmen who make the drawings that machinists use often express the dimensions of objects in *fractional parts* of an inch, such as $\frac{1}{4}$ in., $\frac{3}{8}$ in., $\frac{5}{16}$ in., etc. It becomes necessary, therefore, for machinists, draftsmen, and others to know how to change common fractions to decimals of equal value. For example, if the drawing of a spark plug shows that one of its parts has a length of $\frac{3}{8}$ in., the machinist has to know that $\frac{3}{8}$ in. = .375 in.

A fraction like $\frac{3}{8}$ may be considered a way of indicating the division of 3 by 8. That is, $\frac{3}{8}$ means $3 \div 8$; the line between the 3 and the 8 means "divided by."

To change $\frac{3}{8}$ to a decimal, proceed as follows:

Since $\frac{3}{8}$ means $3 \div 8$, divide 3 by 8. The quotient will be the desired decimal. Before dividing, annex zeros to 3, as shown at the right. The quotient is .375. Hence $\frac{3}{8} = .375$. This answer is easily verified since $.375 = \frac{375}{1000} = \frac{3}{8}$.

```
.375
8)3.000
  24
  60
  56
  40
  40
```

CHANGING FRACTIONS TO DECIMALS

Change each fraction to a decimal. Do the work at the right of each example:

	a	b	c	d
(1)	$\frac{3}{5}$ = .600 / 5 | 3.000	$\frac{5}{8}$ = .625 / 8 | 5.000	$\frac{1}{2}$ = .500 / 2 | 1.000	$\frac{3}{8}$ = .375 / 8 | 3.0040
(2)	$\frac{1}{4}$ = .250 / 4 | 1.000	$\frac{9}{25}$ = .360 / 25 | 9.000	$\frac{1}{8}$ = .125 / 8 | 1.000	$\frac{4}{5}$ = .800 / 5 | 4.000
(3)	$\frac{2}{5}$ = .400 / 5 | 2.000	$\frac{39}{50}$ = .780	$\frac{3}{4}$ = .750 / 4 | 3.000	$\frac{19}{20}$ = .950 / 20 | 19.000
(4)	$\frac{16}{25}$ = .640	$\frac{7}{8}$ = .875	$\frac{7}{20}$ = .350	$\frac{3}{25}$ = .120

The machinist has to know the decimal values of fractions like $\frac{5}{16}$, $\frac{7}{32}$, and $\frac{13}{64}$. Change the following fractions to decimals:

(5)	$\frac{1}{16}$ = .0625	$\frac{15}{16}$ = .9375 / 16 | 15.0000	$\frac{3}{16}$ = .1875	$\frac{11}{16}$ = .6875
(6)	$\frac{5}{16}$ = .3125	$\frac{9}{16}$ = .5625	$\frac{7}{16}$ = .4375	$\frac{13}{16}$ = .8125
(7)	$\frac{7}{32}$ = .21875	$\frac{13}{64}$ = .20315	$\frac{15}{32}$ = .46875	$\frac{35}{64}$ = .54685

1/2 = .5000

(1) Do Improvement Test J2 on Multiplication. See page 144.

In changing fractions to decimals, it often happens that the quotient does not come out exact. For example, in changing $\frac{2}{7}$ to a decimal, the quotient can be carried to as many places as you wish. It becomes necessary, therefore, to decide where to stop. One way to do this is to state that the quotient should be found correct to the nearest hundredth or thousandth.

```
.2857
7)2.0000
 14
 60
 56
 40
 35
 50
 49
  1
```

In the example at the right, the quotient, .2857, has to be carried to 4 places to get a result correct to the nearest thousandth. Rounding off to the nearest thousandth, you get .286. For a result correct to the nearest hundredth, the work should be stopped after obtaining 3 places; in this case the quotient, .285, becomes .29 when rounded off to the nearest hundredth.

In the example at the right, the division could be continued indefinitely. No matter how many quotient figures you obtain, there will always be a remainder. That is, the quotient will never come out exact.

Change to decimals, correct to the nearest thousandth. Write the answer after the fraction. Do the work on a separate sheet of paper:

	a	*b*	*c*	*d*
(2)	$\frac{1}{3}$ = .3333	$\frac{4}{9}$ = .4444	$\frac{5}{7}$ = .7142	$\frac{2}{9}$ = .2222
(3)	$\frac{7}{12}$ = .583	$\frac{2}{13}$ = .154	$\frac{15}{17}$ = .882	$\frac{23}{42}$ = .548
(4)	$\frac{10}{19}$ = .526	$\frac{19}{29}$ = .656	$\frac{10}{23}$ = .434	$\frac{24}{31}$ = .774
(5)	$\frac{2}{3}$ = .666	$\frac{11}{12}$ = .916	$\frac{13}{17}$ = .764	$\frac{28}{37}$ = .755

Change to decimals, correct to the nearest hundredth:

(6)	$\frac{41}{67}$ = .61 $\frac{3}{67}$	$\frac{28}{39}$ = .71 $\frac{31}{39}$	$\frac{31}{45}$ = .68 $\frac{40}{45}$	$\frac{49}{61}$ =
(7)	$\frac{11}{15}$ =	$\frac{5}{12}$ = .41 $\frac{8}{12}$	$\frac{14}{31}$ =	$\frac{31}{59}$ =
(8)	$\frac{13}{18}$ =	$\frac{24}{35}$ =	$\frac{7}{36}$ =	$\frac{15}{28}$ =
(9)	$\frac{11}{13}$ =	$\frac{25}{36}$ =	$\frac{28}{71}$ =	$\frac{39}{43}$ =

For more practice, see rows 2 to 4 on page 160.

DIVIDING BY A DECIMAL

A Useful Principle. Any example in division such as $18 \div 6$ can be expressed in the form of a fraction. For example, $18 \div 6$ may be written as $\frac{18}{6}$. If you multiply both the numerator and denominator of $\frac{18}{6}$ by 10 or by 100, you do not change the value of the fraction. Thus $\frac{18}{6} = \frac{180}{60} = \frac{1800}{600}$. Each of these fractions has the same value, which is 3. If you write these three fractions in the usual division form, you get:

$$6\overline{)18}\ 3 \qquad 60\overline{)180}\ 3 \qquad 600\overline{)1800}\ 3$$

You see that the quotient in each case remains 3; that is, its value remains unchanged. This illustrates a very useful principle, as follows:

If the dividend and divisor are both multiplied by 10, *or by* 100, *or by* 1000, *the quotient remains the same.*

Dividing by a Decimal. Tom saves \$1.25 a week. How many weeks will it take him to save enough to buy a second-hand bicycle for \$15.00?

It is necessary to divide \$15.00 by \$1.25. You do not know how to divide when the divisor is a decimal, as shown in *A*. But if the divisor were a whole number, as shown in *B*, you could divide. If the divisor and the dividend of *A* are each multiplied by 100, you get 125 as a new divisor and 1500 as a new dividend, as shown in *B*. This is an application of the principle stated above. If you then divide the numbers in *B*, the quotient is 12. Hence it will take Tom 12 weeks to save enough money to buy the bicycle.

A	B
$1.25\overline{)15.00}$	$125\overline{)1500}\ 12$

Check the work by multiplying 1.25, the original divisor, by 12. If the product equals 15.00, the original dividend, the work is right.

Copy and divide. First change the divisor to a whole number by multiplying divisor and dividend by 100, as explained above. Check by multiplying:

	a	*b*	*c*	*d*
(1)	\$60.48 ÷ \$.84	\$7.05 ÷ \$2.35	\$59.76 ÷ \$1.66	\$75.24 ÷ \$1.71
(2)	\$19.25 ÷ \$.55	\$9.94 ÷ \$4.97	\$57.98 ÷ \$2.23	\$94.25 ÷ \$7.25
(3)	\$33.44 ÷ \$.38	\$8.16 ÷ \$2.72	\$68.75 ÷ \$1.25	\$76.59 ÷ \$3.33
(4)	\$74.10 ÷ \$.95	\$9.90 ÷ \$1.65	\$84.36 ÷ \$2.28	\$79.98 ÷ \$1.86
(5)	\$29.61 ÷ \$.63	\$9.92 ÷ \$2.48	\$29.75 ÷ \$1.75	\$90.30 ÷ \$6.45
(6)	\$31.85 ÷ \$.49	\$6.56 ÷ \$3.28	\$87.92 ÷ \$3.14	\$92.00 ÷ \$3.68
(7)	\$48.10 ÷ \$.74	\$8.56 ÷ \$2.14	\$75.82 ÷ \$2.23	\$96.84 ÷ \$5.38
(8)	\$24.05 ÷ \$.65	\$9.69 ÷ \$3.23	\$92.48 ÷ \$2.89	\$66.30 ÷ \$1.95

For more practice, see rows 16 to 20 on page 158.

Divide 28.776 by 8.8 and check the work.

First change 8.8 to a whole number by multiplying it by 10; to do this move the decimal point one place to the right, using a caret (∧) to show the new position of the decimal point. Then multiply 28.776 by 10 so that the quotient will remain unchanged; to do this place a caret one place to the right of the decimal point in 28.776. The decimal point in the quotient is placed *directly above* the caret in 28.7∧76. When carets are used to show the new positions of the decimal points, they save the trouble of erasing the original decimal points or of writing the problem over again as was done on page 66.

```
      3.27
8.8∧)28.7∧76
     26 4
     ----
      2 3 7
      1 7 6
      -----
        6 1 6
        6 1 6
        -----
```

In the work at the right, you found the first figure, 3, of the quotient by dividing 287 by 88. It is very important that this first quotient figure be placed in the right position, which is *directly above* the 7 of 287. If 3 had been placed over the 8 of 287, the decimal point in the quotient would be in the wrong place because that would give a quotient of 32.7 instead of the correct quotient of 3.27. Check by multiplying 3.27 by the original divisor 8.8. If the product equals the original dividend, 28.776, the work is right.

In dividing by a decimal, place a caret (∧) as many places to the right of the decimal point in both divisor and dividend as there are decimal places in the divisor. Place the decimal point in the quotient directly above the caret in the dividend.

Copy, divide, and check. Use carets to show the new positions of the decimal points:

	a	*b*	*c*	*d*
(1)	.18)63.90	.87)68.208	.72).29232	.36)204.48
(2)	2.8)96.32	.83)5.8183	5.3)1.2402	48.3)9.8532
(3)	.21)68.25	3.9)198.51	5.7)467.97	36.4)833.56
(4)	.87)9.831	5.5)4.6915	4.8)12.672	.092)3.7076
(5)	.76).9424	2.8)24.248	6.4)4.6848	.078)56.628
(6)	.64)7.424	7.8)4.1886	3.7)239.76	.067)5.5275
(7)	.51)173.91	8.8)45.232	.73)51.246	2.13)134.19

For more practice, see rows 1 to 4 on page 159.

DIVIDING BY A DECIMAL

(1) Do Improvement Test J3 on Multiplication. See page 144.

Copy and divide. Annex enough 0's after the dividend to make the quotient come out exact. Check the work by multiplying:

	a	*b*	*c*	*d*
(2)	.32)13.6	4.5)39.78	3.84)124.8	2.56)15.232
(3)	4.8)3.48	.035)2.212	.448).2576	33.6)1898.4
(4)	.16)7.12	.065).5577	1.12).7224	51.2)212.48
(5)	.64).464	19.2)10.08	1.72).7826	1.48)1.0878
(6)	9.6)8.88	.224)1.176	.155)13.361	.128)1.0304

Copy and divide. Carry the quotient to three decimal places; then round off the result correct to the nearest hundredth. Check the work:

(7)	.35)6.281	.31).5827	.56)7.281	3.6)77.84
(8)	.66)2.168	.89).3196	.97)4.926	4.4)15.87
(9)	.78)7.924	.22).4725	.45)1.491	5.3)11.86
(10)	.89)4.903	.71).4888	.66)1.269	1.6)94.98
(11)	.62)1.893	.48).6521	2.4)3.214	3.2)26.723
(12)	.43)3.626	.83).7924	3.7)1.622	9.5)17.014
(13)	.65)1.827	.57)2.711	4.4)6.081	7.3)14.901
(14)	.34)4.736	2.6)1.825	6.1)33.11	.35)1.7086

For more practice, see rows 5 to 14 on page 159.

Study examples *A* and *B* shown at the right.

In the quotient of example *A*, the 8 of .086 is placed above the 3 of 653 because 653 is the first partial dividend. The decimal point in the quotient is placed directly above the caret in the dividend. This leaves an empty space in the quotient above the 5 of 653; this empty space must be filled with a 0. Hence the quotient is .086 and not .86. Check by multiplying .086 by 7.6.

A	B
.086	640.
7.6$_\wedge$).6$_\wedge$536	.23$_\wedge$)147.20$_\wedge$
608	138
456	92
456	92
	0
	0

In *B*, the original example is .23)147.2. Since the caret in the divisor, .23, is put 2 places to the right of the decimal point, the caret in the dividend, 147.2, must also be put 2 places to the right. Since 147.2 has only 1 decimal place, write a 0 after it to make 2 decimal places; then put the caret after the 0, thus: 147.20$_\wedge$. In *B*, don't forget to bring down the final zero of the dividend, 147.20, so that you will get a final zero in the quotient. The quotient is 640 and not 64. Check by multiplying 640 by .23.

In these exercises the decimal points in the quotients are left out. In some quotients the 0's are also omitted. In each quotient put the decimal point in the right place, supplying 0's if necessary:

	a	*b*	*c*	*d*
(1)	93 1.5)1395	78 .21)1638	315 .065)20475	421 1.33)5599.3
(2)	95 4.4)4180	18 .87)1566	47 21.5)10105	94 3.64).34216
(3)	34 .62)2108	57 9.6).5472	29 7.25)21025	46 3.14).14444
(4)	26 5.9)1534	81 .32)259.2	283 1.75)49525	869 .018).15642
(5)	43 7.5).3225	56 .28)156.8	87 16.5).14355	538 .067).36046

For more practice, see rows 15 to 19 on page 159.

DIAGNOSTIC TEST

Add ex. a:

a

(1)
8.175
2.650
3.225
1.667
4.375
5.133
0.525

Multiply ex. b:

b

40.25
.0625

Divide ex. c:

c

$4.3\overline{)273.91}$

Subtract ex. a:

(2)
$285.63
109.44

Change to a decimal:

$\frac{4}{5}$

Change to a decimal correct to the nearest tenth:

$\frac{17}{33}$

Divide. In ex. b and c, find the quotient correct to the nearest hundredth:

a

(3) $5.8\overline{).3683}$

b

$45\overline{)687}$

c

$32\overline{)177}$

-------- RIGHT OUT OF 9

SCORE -------- LINE H

REMEDIAL EXERCISES

See page 138.

Ex. 1 *a*	p. 52, 1–3	Ex. 2 *a*	p. 150, 1–8	Ex. 3 *a*	p. 159, 15–19
Ex. 1 *b*	p. 157, 6–13	Ex. 2 *b*	p. 160, 1	Ex. 3 *b*	p. 158, 11–15
Ex. 1 *c*	p. 159, 1–4	Ex. 2 *c*	p. 160, 2–4	Ex. 3 *c*	p. 158, 11–15

Improvement Test — Division T1 Time 4 min.

Find each quotient correct to the nearest tenth:

	a	*b*	*c*
(1)	$23\overline{)1164}$	$39\overline{)3410}$	$56\overline{)5230}$
(2)	$74\overline{)1851}$	$88\overline{)5606}$	$65\overline{)2251}$

-------- RIGHT OUT OF 6 SCORE -------- LINE E

MIXED PRACTICE

(3) Write the missing numbers: $1\frac{3}{4}$, $2\frac{1}{2}$, $3\frac{1}{4}$, ------, ------, ------.

(4) The product of $\frac{2}{3}$ and $\frac{3}{8}$ is ------.

(5) The quotient of $\frac{9}{10} \div 27$ is ------.

(6) Which is larger, $\frac{2}{3}$ of 48 or $\frac{3}{4}$ of 44? ----------.

(7) The sum of $3\frac{7}{8}$, $2\frac{1}{2}$, $4\frac{3}{4}$, and $3\frac{5}{8}$ is ------.

(8) The difference between $24\frac{1}{2}$ and $19\frac{5}{6}$ is ------.

-------- RIGHT OUT OF 6 SCORE -------- LINE E

In your study of decimals you learned that $\frac{6}{100}$ may be written as .06. Another short way to write $\frac{6}{100}$ is 6%, which is read 6 *per cent.* *Per cent* means *hundredths.* The sign % means *per cent.*

You now have four different ways of writing hundredths, as shown below:

$\frac{6}{100} = .06 = 6\% =$ 6 per cent

Likewise, $\frac{27}{100} = .27 = 27\% =$ 27 per cent

Any one of these four ways of writing hundredths may be changed to another. For example, if you have 8%, you can change it to $\frac{8}{100}$ or to .08 as the occasion may require.

Write each of the following as a decimal:

	a	*b*	*c*	*d*	*e*
(1)	28% = .28	52% =	78% =	13% =	3% =
(2)	49% =	88% =	7% =	2% =	9% =
(3)	$\frac{6}{100}$ =	$\frac{41}{100}$ =	$\frac{4}{100}$ =	$\frac{26}{100}$ =	$\frac{5}{100}$ =

Write each of the following as a per cent, using the sign %:

(4)	.68 = 68%	.20 =	.09 =	.07 =	.16 =
(5)	.33 =	.60 =	.63 =	.03 =	.06 =
(6)	$\frac{18}{100}$ =	$\frac{2}{100}$ =	$\frac{30}{100}$ =	$\frac{7}{100}$ =	$\frac{28}{100}$ =

Put the right numbers in the spaces, as shown in ex. 7:

(7) 15% of 760 means ___.15___ × ___760___, which equals ___114___.

(8) 56% of 125 means _____ × _____, which equals _____.

(9) 85% of 440 means _____ × _____, which equals _____.

(10) 48% of 275 means _____ × _____, which equals _____.

(11) 8% of 1250 means _____ × _____, which equals _____.

For more practice, see rows 6 to 13 on page 160.

(1) Do Improvement Test T2 on Division. See page 144.

(2) Last June 140 students graduated from the George Washington High School. In the fall, 55% of these students went to college. How many students went to college?

55% of 140 is the same as .55 × 140, or 77. Hence 77 students went to college.

```
 140
 .55
 700
700
77.00
```

(3) The students in the Central High School made $193 by giving a play. If 35% of this money goes to the Athletic Club, how much does that club get?

In percentage problems it is important correctly to point off the number of decimal places in the product. Since per cents are decimals, the product should have as many decimal places as there are in the multiplier and the multiplicand together. In the example at the right, there are 2 decimal places in .35 and 0 decimal places in $193, which makes 2 + 0, or 2 decimal places in .35 and 193 together. Hence there should be 2 decimal places in the product, which gives $67.55 as the answer. In pointing off the number of decimal places in the product, always begin at the right and count toward the left. You see that the Athletic Club gets $67.55.

```
$193
 .35
 965
579
$67.55
```

(4) Mr. Brown borrowed $450 for 1 year from Mr. Alexander. At the end of the year Mr. Brown returned the money to Mr. Alexander with interest at 4%. How much interest did Mr. Brown pay?

The interest is 4% of $450 which equals .04 × $450, or $18.00.

```
$450
 .04
$18.00
```

Find the following. Do the work on a separate sheet of paper. Check the work by going over it again:

	a	*b*	*c*	*d*
(5)	56% of 250	37% of 800	5% of 300	7% of 5500
(6)	65% of 480	15% of 240	8% of 125	5% of 2540
(7)	15% of 440	48% of 725	2% of 350	4% of 1125
(8)	28% of 750	24% of 375	6% of 650	8% of 2375
(9)	69% of $45	48% of $976	59% of $203	12% of $2.50
(10)	87% of $34	29% of $169	67% of $167	32% of $7.75
(11)	48% of $85	76% of $239	36% of $980	45% of $4.40
(12)	19% of $87	15% of $960	43% of $236	72% of $8.50

For more practice, see rows 1 to 6 on page 161.

FINDING RESULTS TO THE NEAREST CENT

(1) Do Improvement Test T3 on Division. See page 144.

Rounding Off Results. Examples *A*, *B*, and *C* show the results of finding 7% of three different amounts. In *A* the answer, \$1.5470, is closer to \$1.55 than it is to \$1.54. Hence it is called \$1.55 *correct to the nearest cent.* In *B* the answer, \$1.5820, is closer to \$1.58 than it is to \$1.59. Hence, *to the nearest cent,* it is called \$1.58. In *C* the result, \$1.5750, is halfway between \$1.57 and \$1.58. In this case it is called \$1.58 *correct to the nearest cent.* When a result is written correct to the nearest cent, it is said to be **rounded off** to the nearest cent.

A	B	C
\$22.10	\$22.60	\$22.50
.07	.07	.07
\$1.5470	\$1.5820	\$1.5750

In rounding off a result to the nearest cent, increase the figure in the second decimal place by 1 if the figure following it is 5 or more; leave the figure in the second decimal place unchanged if the figure following it is less than 5.

Round off each amount to the nearest cent. Write the result after the amount:

	a	*b*	*c*	*d*
(2)	\$1.731	\$2.995	\$99.998	\$6.8448
(3)	\$4.687	\$6.173	\$81.701	\$7.9291
(4)	\$5.183	\$5.289	\$46.096	\$5.6255

---

(5) Mr. Chase bought a railroad ticket for \$78.85 on which he had to pay a tax equal to 10% of the price of the ticket. What was the amount of his tax? How much in all did the ticket cost him?

\$78.85
.10
\$7.8850

The total tax is \$7.885, which becomes \$7.89 when rounded off to the nearest cent. How much in all did Mr. Chase pay for the railroad ticket?

Find the following correct to the nearest cent. Do the work on a separate sheet of paper. Check the work by going over it:

(6)	2% of \$84.77	4% of \$10.55	5% of \$125.36	4% of \$50.75
(7)	9% of \$80.50	8% of \$96.42	3% of \$390.45	7% of \$60.15
(8)	16% of \$34.98	78% of \$75.30	49% of \$167.09	22% of \$150.25
(9)	35% of \$97.80	48% of \$76.18	39% of \$712.34	18% of \$550.25

For more practice, see rows 7 to 13 on page 161.

Improvement Test — Addition B 4 Time 4 min.

	a	*b*	*c*	*d*	*e*
(1)	2679	7653	2136	7269	1294
	5205	9533	6807	6603	2563
	1547	2287	9585	5631	4585
	4750	8135	1076	4590	7303
	3841	9406	2327	3021	4224
	1486	8554	4834	1109	6456
	9762	4042	8435	7719	2818
	3455	1328	1962	1549	7087

-------- RIGHT OUT OF 5 SCORE -------- LINE D

PREPARING FOR SHORT CUTS IN PERCENTAGE

Find the following. Round off the result to the nearest cent. To save time, the multiplicand need not be copied:

	a	*b*	*c*	*d*
(2)	$\frac{3}{4}$ of \$2.65	$\frac{2}{3}$ of \$2.56	$\frac{7}{8}$ of \$7.50	$\frac{5}{6}$ of \$9.72

2.65
3
4)7.95
1.987
\$1.99, *Ans.*

(3)	$\frac{3}{8}$ of \$4.42	$\frac{3}{5}$ of \$5.25	$\frac{3}{4}$ of \$8.11	$\frac{5}{8}$ of \$9.60
(4)	$\frac{1}{2}$ of \$2.95	$\frac{1}{4}$ of \$5.96	$\frac{1}{6}$ of \$4.52	$\frac{1}{8}$ of \$4.56

SHORT CUTS IN PERCENTAGE

You know that $75\% = \frac{75}{100} = \frac{3}{4}$. Hence 75% is the same as $\frac{3}{4}$. To find 75% of \$1.85 you could do the work the long way, as shown at the right, but you can get the same result a short way by finding $\frac{3}{4}$ of \$1.85. When you use $\frac{3}{4}$ instead of 75%, you are using a short cut.

Note that the result is \$1.3875 when the work is done the long way and \$1.387 when done the short way. In the short way, the result would have been \$1.3875 if the quotient had been carried to 1 more decimal place, but this was unnecessary since \$1.387 has enough places to give an answer correct to the nearest cent. Both the long way and the short way give the same answer to the nearest cent, which is \$1.39.

Long Way	Short Way
\$1.85	1.85
.75	3
925	4)5.55
1 295	1.387
\$1.3875	
\$1.39, Ans.	\$1.39, Ans.

Here are some other short cuts. Show that each is true:

$50\% = \frac{1}{2}$	$12\frac{1}{2}\% = \frac{1}{8}$	$62\frac{1}{2}\% = \frac{5}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$	$16\frac{2}{3}\% = \frac{1}{6}$
$25\% = \frac{1}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$

Find the following correct to the nearest cent. Use the short cuts:

	a	*b*	*c*	*d*
(1)	75% of \$2.15	$66\frac{2}{3}\%$ of \$4.95	75% of \$9.10	$66\frac{2}{3}\%$ of 55¢
(2)	25% of \$5.43	$33\frac{1}{3}\%$ of \$2.64	50% of \$7.96	$33\frac{1}{3}\%$ of 61¢
(3)	50% of \$9.97	25% of \$8.32	25% of \$7.41	75% of 38¢
(4)	$37\frac{1}{2}\%$ of \$1.76	$87\frac{1}{2}\%$ of \$2.50	$83\frac{1}{3}\%$ of \$3.75	$66\frac{2}{3}\%$ of 73¢
(5)	$62\frac{1}{2}\%$ of \$2.17	75% of \$9.36	$12\frac{1}{2}\%$ of \$7.45	$16\frac{2}{3}\%$ of 94¢

For more practice, see rows 14 to 20 on page 161.

(1) Do Improvement Test B2 on Addition. See page 141.

Per cents like 15%, 28%, and 75% each have a value less than 1. Per cents like 135%, 217%, and 300% each have a value greater than 1, while 100% equals 1. For example, $135\% = \frac{135}{100} = 1.35$; $300\% = \frac{300}{100} = 3.00 = 3$.

Since $100\% = \frac{100}{100} = 1$, you see that 100% of 12 examples means 1×12 examples, or 12 examples. That is, 100% of a number means the *whole* number. A mark of 100% on a test means that *all* the examples are right. Similarly, $200\% = \frac{200}{100} = 2$. If the present population of a city is 200% of the population 10 yr. ago, it means that the present population is *twice* as large as it was 10 yr. ago.

Write each per cent as a decimal or a whole number:

	a	*b*	*c*	*d*	*e*
(2)	185% =	110% =	250% =	600% =	100% =
(3)	256% =	130% =	345% =	108% =	360% =
(4)	500% =	320% =	112% =	125% =	472% =

Write each decimal or whole number as a per cent:

(5)	1.15 =	2.05 =	1.47 =	2.80 =	6 =
(6)	3.95 =	3.00 =	6.84 =	7.00 =	2 =
(7)	1.82 =	5.30 =	4.17 =	4.00 =	8 =

Find the following. Do the work on a separate sheet of paper:

	a	*b*	*c*	*d*
(8)	217% of \$789	489% of \$654	784% of \$123	125% of \$4160
(9)	326% of \$713	465% of \$578	146% of \$690	200% of \$6000
(10)	389% of \$890	176% of \$534	100% of \$218	300% of \$1750
(11)	300% of \$546	500% of \$396	200% of \$456	240% of \$8500

For more practice, see rows 6 to 13 on page 160 and rows 1 to 4 on page 162.

PER CENTS ENDING IN FRACTIONS

Find the following, correct to the nearest cent. In ex. 1 a, first multiply 560 by 3; then find $\frac{1}{2}$ of 560; be sure to put 280 in the right position:

	a	*b*	*c*
(1)	$3\frac{1}{2}\%$ of \$560	$4\frac{1}{2}\%$ of \$270	$2\frac{1}{2}\%$ of \$184.60
(2)	$8\frac{1}{4}\%$ of \$284	$3\frac{1}{3}\%$ of \$117	$5\frac{1}{4}\%$ of \$378.24
(3)	$4\frac{3}{4}\%$ of \$332	$6\frac{2}{3}\%$ of \$771	$2\frac{3}{4}\%$ of \$550.80
(4)	$5\frac{3}{4}\%$ of \$84.40	$4\frac{1}{2}\%$ of \$57.25	$3\frac{2}{3}\%$ of \$85.75

560
$.03\frac{1}{2}$
16 80
2 80
19.60

For more practice, see rows 5 to 8 on page 162.

MIXED PRACTICE

(1) Do Improvement Test B3 on Addition. See page 141.

Add

(2)
$$\begin{array}{r} 14.316 \\ 1.225 \\ 2.062 \\ 11.750 \\ 3.617 \\ 9.421 \\ 12.055 \\ \underline{6.002} \end{array}$$

Divide

(3) $75\overline{)1222.5}$

Multiply

(4)
$$\begin{array}{r} 89.6 \\ \underline{.075} \end{array}$$

Subtract

(5)
$$\begin{array}{r} 6.375 \\ \underline{2.750} \end{array}$$

Find

(6) $\frac{3}{4}$ of 75

Divide

(7) $6\overline{)8.124}$

Change to a decimal correct to the nearest hundredth

(8) $\frac{31}{57}$

Find

(9) 64% of 675

Find

(10) $37\frac{1}{2}\%$ of \$960

Find

(11) 235% of \$840

Find

(12) $2\frac{1}{2}\%$ of \$350

Multiply

(13)
$$\begin{array}{r} 25 \\ \underline{3\frac{3}{4}} \end{array}$$

-------- RIGHT OUT OF 12

SCORE -------- LINE L

CHANGING FRACTIONS TO PER CENTS

Change each fraction to a decimal; then change the decimal to a per cent:

	a	*b*	*c*
(1)	$\frac{72}{96}$	$\frac{39}{52}$	$\frac{17}{25}$
(2)	$\frac{13}{20}$	$\frac{24}{75}$	$\frac{28}{80}$
(3)	$\frac{23}{25}$	$\frac{37}{74}$	$\frac{34}{85}$
(4)	$\frac{24}{40}$	$\frac{18}{75}$	$\frac{48}{60}$
(5)	$\frac{17}{50}$	$\frac{13}{25}$	$\frac{48}{64}$

$$
\begin{array}{r}
.75 = 75\% \\
96\overline{)72.00} \\
67\,2 \\
\hline
4\,80 \\
4\,80 \\
\hline
\end{array}
$$

For more practice, see rows 1 to 4 on page 163.

Change each fraction to a two-place decimal, writing remainders, if any, in fractional form. Then change the decimal to a per cent. Do the work as in ex. 1 a:

	a	*b*	*c*	*d*
(1)	$\frac{2}{75}$ $\quad .02\frac{2}{3} = 2\frac{2}{3}\%$ $75\overline{)2.00}$ $\underline{1\,50}$ 50	$\frac{17}{60}$	$\frac{9}{40}$	$\frac{35}{36}$
(2)	$\frac{13}{80}$	$\frac{2}{15}$	$\frac{13}{16}$	$\frac{26}{32}$

COMPARING NUMBERS

Write a fraction in each space to tell what part the smaller number is of the larger. Reduce fractions to lowest terms. In ex. 3 a think: "6 is $\frac{6}{15}$ of 15; $\frac{6}{15} = \frac{2}{5}$."

(3)	6 is $\frac{2}{5}$ of 15	26 is ___ of 39	63 is ___ of 70	28 is ___ of 35
(4)	9 is ___ of 18	22 is ___ of 44	12 is ___ of 60	30 is ___ of 36
(5)	6 is ___ of 10	27 is ___ of 36	17 is ___ of 25	28 is ___ of 32
(6)	6 is ___ of 16	20 is ___ of 32	36 is ___ of 48	24 is ___ of 64
(7)	8 is ___ of 12	14 is ___ of 25	18 is ___ of 20	18 is ___ of 27
(8)	5 is ___ of 15	18 is ___ of 45	25 is ___ of 40	36 is ___ of 90
(9)	9 is ___ of 12	24 is ___ of 36	28 is ___ of 63	45 is ___ of 50

For more practice, see rows 1 to 4 on page 163.

Improvement Test — Subtraction E 4 Time 4 min.

	a	*b*	*c*	*d*
(1)	521783 219407	600501 299694	313815 249716	427444 138676
(2)	739684 480995	949928 589979	100000 52838	167353 94624
(3)	500005 399999	214116 183927	135085 51948	521446 442316

-------- RIGHT OUT OF 12 SCORE -------- LINE L

COMPARING NUMBERS BY PER CENTS

The Giants won 17 games out of 25. What per cent of the games did they win?

To find what per cent 17 is of 25, proceed as follows:

(1) Find what fractional part 17 is of 25. 17 is $\frac{17}{25}$ of 25.
(2) Change $\frac{17}{25}$ to a two-place decimal, as shown at the right. You find that $\frac{17}{25} = .68$.
(3) Change .68 to a per cent. .68 = 68%. Hence 17 is 68% of 25.
(4) Check the work by finding 68% of 25; if the result equals 17, the work is correct.

```
      .68 = 68%
25)17.00
   150
   200
   200
```

Check

```
  25
 .68
 200
 150
17.00
```

The Giants won 68% of the 25 games.

To find what per cent one number is of another, first find what fractional part the one number is of the other; then change this fraction to a two-place decimal and change the decimal to a per cent.

Find each per cent mentally, writing the answer in the space:

(4)	10 is ____% of 20	12 is ____% of 48	40 is ____% of 160
(5)	15 is ____% of 45	45 is ____% of 60	50 is ____% of 150
(6)	20 is ____% of 30	10 is ____% of 50	50 is ____% of 200

Find each per cent as shown below. Check the work on a separate sheet of paper:

	a	*b*	*c*
(1)	27 is _60_% of 45	37 is _____% of 50	81 is _____% of 450
(2)	45 is $62\frac{1}{2}$% of 72	49 is _____% of 56	35 is _____% of 112

$$\begin{array}{r} .60 \\ 45\overline{)27.00} \\ 27\,0 \\ \hline \end{array}$$

$$\begin{array}{r} .62\frac{1}{2} \\ 72\overline{)45.00} \\ 43\,2 \\ \hline 1\,80 \\ 1\,44 \\ \hline 36 \end{array}$$

Find each per cent, writing the answer in the space. Do the work on a separate sheet of paper and check:

(3)	44 is ____% of 55	49 is ____% of 140	111 is ____% of 148
(4)	12 is ____% of 75	72 is ____% of 150	98 is ____% of 350
(5)	32 is ____% of 25	52 is ____% of 80	63 is ____% of 144
(6)	49 is ____% of 98	20 is ____% of 75	255 is ____% of 600
(7)	57 is ____% of 60	36 is ____% of 24	175 is ____% of 350
(8)	35 is ____% of 56	187 is ____% of 68	168 is ____% of 120
(9)	42 is ____% of 48	150 is ____% of 50	135 is ____% of 270

For more practice, see rows 1 to 4 on page 163.

1. Do Improvement Test E2 on Subtraction. See page 143.

2. Last year Mr. Brown sold 72 cars. This year he sold 90 cars. This year's sales are what per cent *more* than last year's sales?

Mr. Brown sold 18 more cars this year than last. (90 − 72 = 18.) You find that 18 is 25% of 72, hence this year's sales of 90 cars are 25% more than last year's sales. This is an increase of 25% over last year's sales.

3. Mr. Allen sold 60 cars last year but this year he sold only 54 cars. This year's sales are what per cent less than last year's sales?

He sold 6 cars less this year. 6 is 10% of 60, hence this year's sales are 10% below last year's sales.

4. If the population of a city increases from 17,500 to 21,875 in 10 years, what is the per cent of increase in that period? If the population decreases from 21,875 to 17,500 in 10 years, what is the per cent of decrease? Note that the amount of increase in the first case equals the amount of decrease in the second case, but the per cent of increase is greater than the per cent of decrease. Explain why.

To find the per cent of increase, first find the actual increase; then find what per cent this increase is of the smaller number. To find the per cent of decrease, first find the actual decrease; then find what per cent this decrease is of the larger number.

The numbers of pupils in several schools for two different years are given below. Find the per cent of increase or decrease in the pupils in each school. Put the sign + before answers showing increases and the sign − before those showing decreases. Do the work on a separate sheet of paper:

	School	Number of Pupils		Per Cent of Increase or Decrease
		5 Yr. Ago	Now	
5.	High	850	1088	------
6.	Lane	240	276	------
7.	Park	170	153	------
8.	Hill	192	216	------
9.	Rice	180	189	------

	School	Number of Pupils		Per Cent of Increase or Decrease
		5 Yr. Ago	Now	
10.	Fox	210	147	------
11.	Case	225	189	------
12.	Ford	175	196	------
13.	Cort	312	364	------
14.	Burt	400	390	------

For more practice, see rows 13 to 18 on page 163.

Mr. Brown received a commission of \$787.50 for selling a house. This commission was 5% of the price at which he sold the house. What price did he get for the house?

5% of the price = \$787.50
1% of the price = \$787.50 ÷ 5, or \$157.50
100% of the price = 100 × \$157.50, or \$15,750

Since 100% of the price equals the entire price, Mr. Brown sold the house for \$15,750. To check the work, find 5% of \$15,750, which equals \$787.50.

Write the correct number in each space. Check the work:

	a	*b*	*c*
(1)	42 is 5% of ------	30 is 8% of ------	33 is 6% of ------
(2)	27 is 3% of ------	37 is 4% of ------	42 is 7% of ------

Copy and work the following exercises. Check the work:

(3)	84 is 24% of ------	57 is 15% of ------	96 is 75% of ------
(4)	91 is 65% of ------	76 is 16% of ------	98 is 28% of ------
(5)	16 is 4% of ------	63 is 18% of ------	42 is 12% of ------
(6)	20 is 5% of ------	48 is 15% of ------	66 is 15% of ------
(7)	15 is 3% of ------	75 is 15% of ------	168 is 35% of ------
(8)	12 is 8% of ------	16 is $12\frac{1}{2}$% of ------	231 is 33% of ------

For more practice, see rows 9 to 12 on page 163.

(1) Do Improvement Test E3 on Subtraction. See page 143.

The Star Manufacturing Company makes automobile parts. Last year the total sales of the company were \$72,400 and 6.3% of this amount was profit. What was the company's profit last year?

First change 6.3% to a decimal. To do this, write 6.3% as $\frac{6.3}{100}$. Then divide 6.3 by 100 by moving the decimal point two places to the left, which gives .063. Hence 6.3% = .063. To find the profit, multiply \$72,400 by .063.

To change a per cent like 27.3% *to a decimal, drop the sign % and move the decimal point two places to the left.*

Thus, 27.3% = .273; 5.18% = .0518; 0.3% = .003.

To change a decimal to a per cent, move the decimal point two places to the right and annex the sign %.

Thus, .097 = 9.7%; .1463 = 14.63%; .0009 = 0.09%.

In ex. 2, change the per cents to decimals; in ex. 3, change the decimals to per cents:

	a	*b*	*c*	*d*	*e*
(2)	12.8% =	0.7% =	0.01% =	8.4% =	3.2% =
(3)	.063 =	.0545 =	.003 =	.164 =	.215 =

Copy; then find the results, correct to the nearest cent:

	a	*b*	*c*	*d*
(4)	9.8% of \$438	3.3% of \$844	4.1% of \$918	6.2% of \$143
(5)	7.2% of \$962	5.7% of \$295	2.8% of \$653	1.4% of \$822
(6)	0.6% of \$125	0.9% of \$435	0.5% of \$347	2.3% of \$235
(7)	1.8% of \$932	1.5% of \$728	5.3% of \$678	4.5% of \$175
(8)	3.4% of \$615	1.4% of \$465	0.8% of \$466	0.6% of \$632
(9)	2.6% of \$826	2.5% of \$625	3.8% of \$545	0.2% of \$483
(10)	4.2% of \$159	1.3% of \$900	0.6% of \$987	1.5% of \$925

For more practice, see rows 6 to 13 on page 160 and rows 9 to 12 on page 162.

Improvement Test — Multiplication K1 Time 4 min.

(1) $\begin{array}{r} 2683 \\ \times\ 904 \\ \hline \end{array}$ $\begin{array}{r} 5170 \\ \times\ 725 \\ \hline \end{array}$

(2) $\begin{array}{r} 2178 \\ \times\ 376 \\ \hline \end{array}$ $\begin{array}{r} 4326 \\ \times\ 861 \\ \hline \end{array}$

-------- RIGHT OUT OF 4 SCORE -------- LINE C

MIXED PRACTICE

Divide

(3) $98 \div 12\frac{1}{4} =$

Subtract

(4) $\begin{array}{r} 9.375 \\ -\ 4.75 \\ \hline \end{array}$

Find

(5) $33\frac{1}{3}\%$ of \$2874

Multiply

(6) $\begin{array}{r} \$38.73 \\ \times\ 125 \\ \hline \end{array}$

Divide

(7) $285\overline{)178980}$

Add

(8) $\begin{array}{r} \frac{7}{8} \\ \frac{1}{2} \\ \frac{9}{16} \\ \frac{3}{4} \\ \hline \end{array}$

-------- RIGHT OUT OF 6 SCORE -------- LINE E

(1) Do Improvement Test K2 on Multiplication. See page 144.

Find, correct to the nearest tenth of 1%, what per cent the first number is of the second. Carry the quotient to four decimal places; then change it to a per cent:

	a	*b*	*c*	*d*
(2)	86, 97	25, 77	31, 84	118, 149

88.7%, *Ans.*

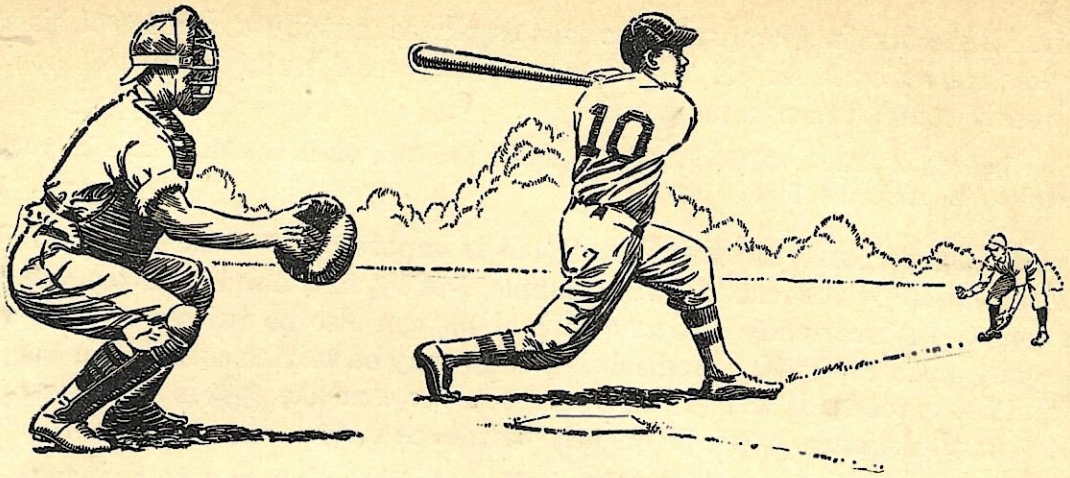
```
    .8865
97)86.0000
   77 6
   ----
    8 40
    7 76
    ----
     640
     582
     ---
      580
      485
      ---
```

(3)	22, 67	17, 71	58, 63	196, 59

Copy and work the following on a separate sheet of paper:

(4)	10, 39	28, 31	40, 41	130, 175
(5)	16, 83	19, 27	37, 47	125, 186
(6)	29, 41	11, 38	83, 95	152, 173
(7)	84, 93	15, 19	14, 91	111, 165

For more practice, see rows 5 to 8 on page 163.



How to Find Batting Averages. In the tables below, A.B. represents the number of times the player has been at bat, H. represents the number of hits he has made, P.C. means per cent. To find the batting average, find the per cent that H. is of A.B. It is the practice in newspapers to express the per cents as decimals, without the sign %. Thus, in ex. 1, the batting average of Williams is .388, which means 38.8%.

Find, to the nearest tenth of 1%, the batting averages of the following players:

AMERICAN LEAGUE

	PLAYER	A.B.	H.	P.C.
1.	Williams	420	163	*.388*
2.	Mantle	474	173	----
3.	Woodling	429	138	----
4.	Boyd	485	154	----
5.	Fox	619	196	----

NATIONAL LEAGUE

	PLAYER	A.B.	H.	P.C.
6.	Musial	502	176	----
7.	Mays	585	195	----
8.	Aaron	615	198	----
9.	Robinson	611	197	----
10.	Groat	501	158	----

11. The standing of a baseball team is found by dividing the number of games the team has won by the number of games that it has played. The result is carried to 4 decimal places and then rounded off to the nearest thousandth, as in finding batting averages. Suppose that Team A has won 56 games out of 99 games and that Team B has won 52 games out of 96 games; find the standing of each team, correct to the nearest thousandth. Which team has the higher standing?

Mr. Allen works in a factory where each employee contributes $\frac{1}{4}$ of 1% of his weekly salary to a recreation fund. If Mr. Allen receives a weekly salary of \$84.00, how much is his weekly contribution to the fund?

First find 1% of \$84.00, which is \$.84. Then find $\frac{1}{4}$ of \$.84, which is \$.21. Mr. Allen contributes 21¢ a week to the recreation fund.

In writing per cents smaller than 1%, it is important to avoid the possibility of misinterpretation by the reader. For example, $\frac{1}{2}$ of 1% may also be written as 0.5% but this is sometimes incorrectly read as 5%; $\frac{1}{2}$ of 1% can also be written as $\frac{1}{2}$% but this, too, is easily misinterpreted. Similarly, $\frac{1}{10}$ of 1% may be written as 0.1% but one has to be careful not to read this as 1%. In business, a per cent like $\frac{3}{10}$% is usually written as $\frac{3}{10}$ of 1%, but in science $\frac{3}{10}$% is frequently written as 0.3%.

If you wish to find 0.25% of 2400, it is probably safest first to change 0.25% to .0025 and then to multiply 2400 by .0025. If you wish to find $\frac{1}{4}$ of 1% of 2400, which is another way of stating the same problem, it is better first to find 1% of 2400 and then to take $\frac{1}{4}$ of this result.

Copy these problems. Find the results correct to the nearest cent:

	a	*b*	*c*
(1)	$\frac{1}{2}$ of 1% of \$680	$\frac{1}{4}$ of 1% of \$3830	0.1% of \$738
(2)	$\frac{1}{3}$ of 1% of \$476	$\frac{1}{10}$ of 1% of \$3200	0.8% of \$832
(3)	$\frac{1}{5}$ of 1% of \$925	$\frac{1}{8}$ of 1% of \$6800	0.2% of \$186
(4)	$\frac{1}{4}$ of 1% of \$540	$\frac{1}{2}$ of 1% of \$377.50	0.6% of \$389
(5)	$\frac{3}{4}$ of 1% of \$125	$\frac{1}{10}$ of 1% of \$920.10	0.5% of \$490
(6)	$\frac{3}{5}$ of 1% of \$525	$\frac{2}{3}$ of 1% of \$425.10	0.3% of \$780
(7)	$\frac{1}{4}$ of 1% of \$250	$\frac{1}{4}$ of 1% of \$185.60	0.4% of \$669
(8)	$\frac{2}{3}$ of 1% of \$650	$\frac{1}{5}$ of 1% of \$235.15	0.1% of \$328
(9)	$\frac{1}{2}$ of 1% of \$259	$\frac{3}{4}$ of 1% of \$185.50	0.8% of \$735
(10)	$\frac{1}{10}$ of 1% of \$426	$\frac{3}{5}$ of 1% of \$265.25	0.7% of \$817
(11)	$\frac{1}{10}$ of 1% of \$855	$\frac{1}{2}$ of 1% of \$382.50	0.5% of \$575

For more practice, see rows 13 to 16 on page 162.

(1) Do Improvement Test K3 on Multiplication. See page 144.

(2) At the Sports Shop tennis rackets were sold at the end of the summer at a discount of 35%. If the regular price of a tennis racket was \$7.80, what was the price after deducting the discount?

35% of \$7.80 = \$2.73, the discount. The tennis racket would cost \$7.80 − \$2.73, or \$5.07. \$5.07 is called the *net price* of the tennis racket.

(3) At a sale, footballs that regularly sold at \$8.50 were sold at "$\frac{1}{4}$ off." Find, to the nearest cent, the price of one of these footballs. What per cent discount is the same as "$\frac{1}{4}$ off"?

(4) At a sale Tom paid \$6.00 for a football. If the regular price of the football was \$7.50, what per cent discount did Tom receive?

(5) At a sale Mrs. Adams bought a dress for \$16.00, which was 20% less than the regular price. What was the regular price of the dress? At the same sale Mrs. Allen bought a dress for \$18.00, which was $\frac{1}{3}$ less than the regular price. What was the regular price of Mrs. Allen's dress?

The marked prices of articles and the rates of discount are given below. Find, to the nearest cent, the amount of discount and the net price:

	a	*b*	*c*	*d*
(6)	\$8 less 20%	\$9 less $16\frac{2}{3}$%	\$150 less $33\frac{1}{3}$%	\$200 less $12\frac{1}{2}$%
(7)	\$6 less 15%	\$6 less $33\frac{1}{3}$%	\$120 less 35%	\$6.80 less 10%
(8)	\$5 less 25%	\$8 less $37\frac{1}{2}$%	\$135 less 14%	\$25.60 less 18%
(9)	\$7 less 12%	\$4 less $12\frac{1}{2}$%	\$724 less $12\frac{1}{2}$%	\$17.50 less 15%
(10)	\$9 less 15%	\$6 less $37\frac{1}{2}$%	\$960 less $37\frac{1}{2}$%	\$24.70 less $12\frac{1}{2}$%
(11)	\$4 less 20%	\$3 less $16\frac{2}{3}$%	\$289 less $33\frac{1}{3}$%	\$654 less $33\frac{1}{3}$%
(12)	\$7 less 25%	\$2 less $12\frac{1}{2}$%	\$125 less $66\frac{2}{3}$%	\$12.75 less 25%
(13)	\$9 less 20%	\$7 less $33\frac{1}{3}$%	\$240 less $12\frac{1}{2}$%	\$18.95 less 10%
(14)	\$5 less 12%	\$8 less $16\frac{2}{3}$%	\$175 less $33\frac{1}{3}$%	\$12.50 less 15%
(15)	\$3 less 15%	\$5 less $12\frac{1}{2}$%	\$120 less $37\frac{1}{2}$%	\$19.98 less 20%
(16)	\$4 less 18%	\$6 less $16\frac{2}{3}$%	\$350 less $62\frac{1}{2}$%	\$27.95 less 30%

DIAGNOSTIC TEST

Find the following. In ex. 2 give results to the nearest cent:

	a	*b*	*c*
(1)	65% of \$470	$1\frac{3}{4}\%$ of \$500	3.6% of \$450
(2)	15% of \$5.75	75% of \$4.25	$\frac{1}{2}$ of 1% of \$975
(3)	$\frac{11}{25}$ is equal to ------%	45 is ------% of 120	29 is 5% of ------
	Find	Find the per cent of increase	Find to nearest 0.1%
(4)	108% of 325	If \$45 increases to \$54.	8 is ------% of 90

-------- RIGHT OUT OF 12

SCORE -------- LINE L

REMEDIAL EXERCISES

See page 138.

Ex. 1 *a*	p. 161, 1–6	Ex. 2 *b*	p. 161, 14–18	Ex. 3 *c*	p. 163, 9–12
Ex. 1 *b*	p. 162, 5–8	Ex. 2 *c*	p. 162, 13–16	Ex. 4 *a*	p. 162, 1–4
Ex. 1 *c*	p. 162, 9–12	Ex. 3 *a*	p. 163, 1–4	Ex. 4 *b*	p. 163, 13–18
Ex. 2 *a*	p. 161, 7–13	Ex. 3 *b*	p. 163, 1–4	Ex. 4 *c*	p. 163, 5–8

Improvement Test — Division U1 Time 4 min.

Find each quotient correct to the nearest tenth:

	a	*b*	*c*
(1)	$93\overline{)2384}$	$28\overline{)1699}$	$42\overline{)2707}$
(2)	$78\overline{)5923}$	$26\overline{)2400}$	$84\overline{)6365}$

-------- RIGHT OUT OF 6 SCORE -------- LINE E

MIXED PRACTICE

(3) $33\frac{1}{3}\%$ of a number is the same as _____ of it.

(4) A discount of 20% on \$395 is \$_____.

(5) The fraction $\frac{17}{50}$ changed to a per cent is _____%.

(6) 3.7% of \$7.50 to the nearest cent is \$_____.

(7) \$18 is 24% of \$_____. \$9 is 20% of \$_____.

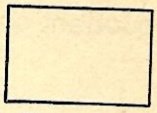
(8) The number that is 25% less than 84 is _____.

(9) An increase of \$15 on a salary of \$75 is an increase of _____%.

-------- RIGHT OUT OF 8 SCORE -------- LINE G

AREA OF A RECTANGLE

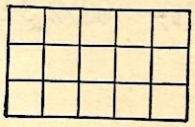
A four-sided figure having four square corners is called a *rectangle.* Each of the square corners forms a *right angle.* Many common objects, like doors and windows, are shaped like rectangles. In the rectangle at the right, two of the sides are longer than the other two sides. If all four sides of a rectangle are equal, the figure is a *square.* A square whose sides are 1 in. long is called a *square inch.*



RECTANGLE

SQUARE

The figure at the right represents a rectangle 5 in. long and 3 in. wide. By dividing it into square inches, you find it contains 3×5, or 15, sq. in. Hence the area of this rectangle is 15 sq. in.



Area of rectangle = length × width

Letting A stand for the area, l for the length, and w for the width, you can write the above rule as a *formula,* thus:

$$A = lw$$

A *formula* is a short way of writing a rule in which letters represent words. When letters are written side by side in a formula, it is understood that they are to be multiplied. Thus, lw means $l \times w$; likewise, $5\,abc$ means $5 \times a \times b \times c$.

Using the formula, work these problems on a separate sheet of paper:

(1) Tom wants to have a vegetable garden as large as possible. His father has 3 plots and will let Tom use any one of them. The dimensions of the plots are 18 ft. by 24 ft., 20 ft. by 20 ft., and 10 ft. by 36 ft. Which plot should Tom choose?

(2) Mr. Weston is building a new garage. At \$2.25 a square yard, how much will it cost to lay the cement floor, which will be 27 ft. 6 in. by 18 ft.? 1 sq. yd. = 9 sq. ft.

In finding the area of a rectangle, express the length and the width in the same unit before multiplying. In this problem, change 27 ft. 6 in. to $27\frac{1}{2}$ ft. Then since $A = lw$, the area is $27\frac{1}{2} \times 18$, or 495, sq. ft. Change the square feet to square yards.

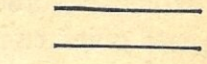
(3) A floor 21 ft. by 18 ft. is to be covered with linoleum that comes in strips 6 ft. wide. Draw a plan of the room to show how the strips will be laid to avoid waste. Also find the cost of the linoleum at \$1.50 per square yard.

Find the area of each rectangle in square feet, writing the answer in the space:

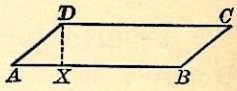
	a	*b*	*c*
(4)	9 ft. × 9 ft. =	4 ft. 2 in. × 9 in. =	18 in. × $5\frac{1}{3}$ ft. =
(5)	3 yd. × 2 yd. =	9 ft. 3 in. × 8 ft. =	27 in. × $5\frac{1}{2}$ ft. =
(6)	6 ft. × 7.5 ft. =	3 ft. 6 in. × 6 in. =	$4\frac{1}{2}$ ft. × $4\frac{1}{2}$ ft. =

(1) Do Improvement Test U2 on Division. See page 144.

Lines like those at the right are called *parallel lines* because they are always the same distance apart.



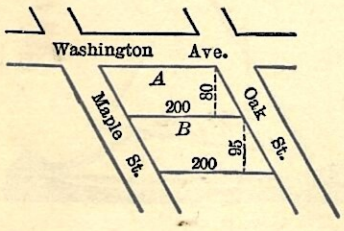
The figure $ABCD$ at the right is called a *parallelogram* because its opposite sides are *parallel;* sides AB and DC are parallel; likewise, sides AD and BC are parallel. The side AB, on which the parallelogram stands, is its *base* and the dotted line DX, perpendicular to the base at X, is its *height.* It is easily shown that the area of a parallelogram equals its base times its height; this rule can be written as a formula, as follows:



$$A = bh$$

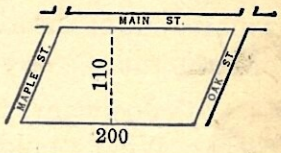
Work these problems on a separate sheet of paper. Put the right word or number in each space:

(2) Mr. Brown bought Lots A and B shown at the left. These lots lie between Maple St. and Oak St., which are parallel streets. Each lot has the shape of a _____. The dimensions of each lot are marked on it. Lot A has a base of _____ ft. and a height of _____ ft.; its area is _____ sq. ft. Lot B has a base of _____ ft. and a height of _____ ft.; its area is _____ sq. ft.



(3) In ex. 2, do Lots A and B together equal about $\frac{1}{2}$ acre, $\frac{3}{4}$ acre, or 1 acre? _____ 1 acre = 43,560 sq. ft.

(4) Mr. Smith bought a lot on Main Street. It was shaped like a parallelogram, with a base of 200 ft. and a height of 110 ft. Mr. Smith's lot contains _____ sq. ft., or about _____ acre.

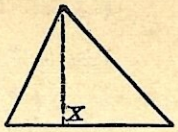


Find the area of each parallelogram in square feet, writing the answer on the line:

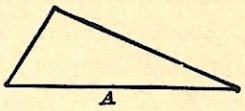
	a			*b*			*c*		
	Base	Height	Area	Base	Height	Area	Base	Height	Area
(5)	8 ft.	5 ft.	_____	16 yd.	14 yd.	_____	8 ft. 6 in.	7 ft.	_____
(6)	9 ft.	7 ft.	_____	14 yd.	10 yd.	_____	6 ft. 9 in.	6 ft.	_____
(7)	5 yd.	3 yd.	_____	21 yd.	18 yd.	_____	4 ft. 4 in.	3 ft.	_____
(8)	7 yd.	4 yd.	_____	12 yd.	$9\frac{1}{2}$ yd.	_____	9 ft. 8 in.	6 ft.	_____

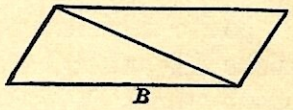
AREA OF A TRIANGLE

A *triangle* is a three-sided figure. The *base* of a triangle is the side on which it stands and its *height* is the perpendicular distance from its highest point to its base. In the triangle at the right the dotted line is the height.



From a piece of cardboard cut two triangles exactly like *A* and place them together to form *B*. You see that *B* is a parallelogram made from 2 equal triangles; hence each triangle is $\frac{1}{2}$ the parallelogram. Since the area of the parallelogram equals its base times its height, the area of each triangle is $\frac{1}{2}$ as large.



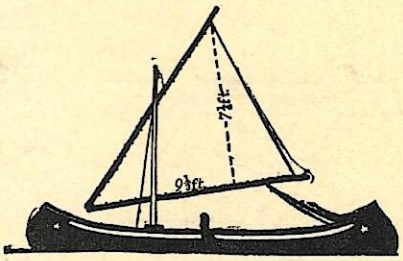


$$\textit{Area of triangle} = \tfrac{1}{2} \times \textit{base} \times \textit{height}$$

This gives the formula: $A = \frac{1}{2}bh$

Work these problems on a separate sheet of paper:

(1) Joe has a triangular sail on his canoe, as shown at the right. The base of the sail is $9\frac{1}{3}$ ft. and the height is $7\frac{1}{4}$ ft. How many square feet are there in the area of the sail?



(2) Joe has another triangular sail that has a height of 8 ft. and a base of $8\frac{3}{4}$ ft. How many square feet larger or smaller is the area of this sail than the one in ex. 1?

(3) In a gentle breeze the wind pushes against a sail with a force of $1\frac{1}{2}$ lb. for each square foot of the sail's area. Find, in pounds, the force of a gentle breeze against each of Joe's sails.

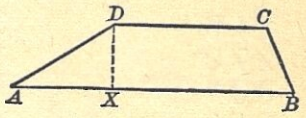
(4) Joe wants to make a smaller triangular sail with a base of 6 ft. and a height of $5\frac{1}{2}$ ft. What would be the force of a strong wind against this sail if such a wind pushes against a sail with a force of $3\frac{1}{2}$ lb. per square foot?

Find the areas of triangles with these dimensions and put the answers in the spaces:

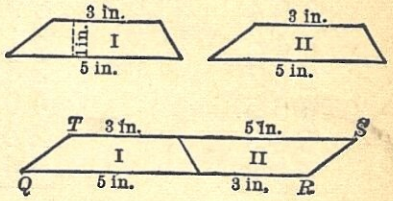
	a			*b*			*c*		
	BASE	HEIGHT	AREA	BASE	HEIGHT	AREA	BASE	HEIGHT	AREA
(5)	7 ft.	5 ft.	------	15 ft.	10 ft.	------	6 yd.	4 yd.	------
(6)	6 ft.	$4\frac{1}{2}$ ft.	------	12 ft.	$8\frac{1}{2}$ ft.	------	3 yd.	$1\frac{1}{2}$ yd.	------
(7)	4 ft.	3 ft.	------	18 ft.	14 ft.	------	8 yd.	5 yd.	------

(1) Do Improvement Test U3 on Division. See page 144.

The figure at the right is a *trapezoid.* In a trapezoid two opposite sides are parallel while the other two sides are not parallel. Thus, AB and DC are parallel but AD and BC are not parallel. The parallel sides AB and DC are called the *bases* and the perpendicular DX is the *height.*



From cardboard cut two equal trapezoids like I and II. If you place I and II end to end, as shown at the right, you get a long parallelogram $QRST$ whose height is 1 in. and whose base is QR, which equals 5 in. + 3 in., or 8 in. The area of $QRST$ equals its height times QR. Each trapezoid is half of $QRST$, hence the area of one trapezoid is half its height times QR. But QR equals 5 in. + 3 in., which is the sum of the upper and the lower bases of each trapezoid. Hence we obtain this rule:



$$\textit{Area of trapezoid} = \tfrac{1}{2} \times \textit{height} \times (\textit{sum of bases})$$

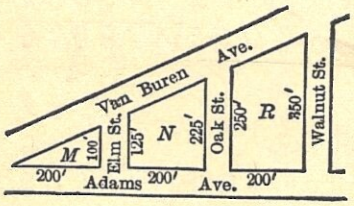
Letting h = height, a = lower base, and b = upper base, we have this formula:

$$A = \tfrac{1}{2}h(a + b)$$

Suppose that the bases of a trapezoid are 15 in. and 21 in. and that the height is 10 in. Then the area of the trapezoid equals $\frac{1}{2} \times 10 \times (15 + 21)$, or 180, sq. in.

Put the right word or number in each space:

(2) On the map at the left Lot N has the shape of a _____. Lot R has the shape of a _____. Lot M has the shape of a _____. The area of Lot M is _____ sq. ft.



(3) The bases of Lot N have lengths of _____ ft. and _____ ft. The height of Lot N is _____ ft. The area of Lot N is _____ sq. ft.

(4) The bases of Lot R have lengths of _____ ft. and _____ ft. The height of Lot R is _____ ft. The area of Lot R is _____ sq. ft.

Find the area of each trapezoid and write the result on the line:

	Bases	Height	Area	Bases	Height	Area	Bases	Height	Area
(5)	3′, 4′	4′	_____	4′, 6′	3′	_____	10′, 7′	6′	_____
(6)	2′, 3′	2′	_____	5′, 7′	6′	_____	12′, 10′	8′	_____
(7)	7′, 5′	3′	_____	8′, 6′	4′	_____	15′, 12′	9′	_____

Improvement Test — Addition C1

Time 4 min.

	a	*b*	*c*	*d*	*e*
(1)	119	372	805	923	312
	270	145	462	280	433
	831	734	924	401	275
	158	452	731	157	651
	384	697	915	410	435
	546	513	143	588	562
	207	286	667	935	409
	665	402	685	279	423
	383	267	430	692	630
	412	445	163	143	238

-------- RIGHT OUT OF 5 SCORE -------- LINE D

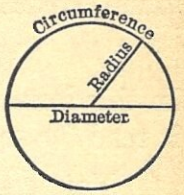
Decimals. Put the decimal point in the right place in each quotient, supplying 0's when necessary:

	a	*b*	*c*	*d*
(2)	49 / 8.5)41.65	808 / .56)45.248	63 / 49)3.087	857 / .059)50.563
(3)	43 / .97)41.71	923 / 7.4).68302	508 / 4.6)23368	634 / 7.55)478670
(4)	98 / 23).2254	562 / 6.6)370.92	333 / .76)253.08	483 / 90.7).438081
(5)	93 / 46).4278	405 / 3.7)14.985	537 / 4.8)2577.6	246 / 28.4)69.864
(6)	65 / .85)55.25	461 / 8.6)39646	209 / .97).20273	861 / .029).24969

For more practice, see rows 15 to 19 on page 159.

(1) Do Improvement Test C2 on Addition. See page 142.

The figure at the right shows the *radius*, *diameter*, and *circumference* of a circle. A line drawn through the center, with its ends on the circle, is a *diameter*. A *radius* is drawn from the center to a point on the circle and equals one half the diameter. The distance around a circle is its *circumference*. The circumference of a circle is always 3.14 times as long as its diameter. The value $3\frac{1}{7}$ is sometimes used instead of 3.14. The Greek letter π (pronounced "pie") stands for 3.14 or $3\frac{1}{7}$. Since you can find the circumference (C) of any circle by multiplying its diameter (D) by π, you can make this formula:



$$C = \pi D$$

Since a diameter equals twice the radius, you may use $2r$ for D, which gives this second formula for C:

$$C = 2\pi r$$

If a bicycle wheel has a diameter of 28 in., its circumference = $3\frac{1}{7} \times 28$ in., or 88 in.

Work these problems on a separate sheet of paper. Let $\pi = 3\frac{1}{7}$:

(2) There is a giant redwood tree in the Sequoia National Park that has a diameter of 37.3 ft. at its base. About how far, to the nearest foot, would a person have to walk to encircle this great tree?

(3) The running track at the George Washington High School has the shape of a circle with a diameter of 140 ft. How many times must one run around this track to cover a mile? 5280 ft. = 1 mi.

(4) The value of π, correct to 5 decimal places, is 3.14159. This value is used only in connection with very precise measurements of the diameter. If 3.14159 is rounded off to 4 decimal places, it becomes _____; if rounded off to 3 decimal places, it becomes _____; if rounded off to 2 decimal places, it becomes _____.

Find, to the nearest tenth of a unit, the circumferences of these circles. Use $\pi = 3.14$:

	a		*b*		*c*
	Radius	Circumference	Radius	Circumference	Radius	Circumference
(5)	6.0 in.	_____	80 ft.	_____	$9\frac{1}{2}$ ft.	_____
(6)	7.5 in.	_____	65 ft.	_____	$6\frac{3}{4}$ ft.	_____
(7)	11.0 in.	_____	23 ft.	_____	72 yd.	_____
(8)	17.5 in.	_____	35 ft.	_____	25 yd.	_____

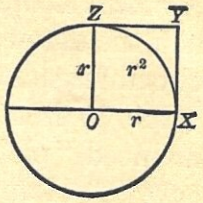
AREA OF A CIRCLE

(1) Do Improvement Test C2 on Addition. See page 142.

(2) If you have studied geometry, you have learned that the area of a circle may be found by using the following formula:

$$A = \pi r^2$$

In this formula r stands for the *radius* of the circle; r^2 means $r \times r$ and is read "r square."



You can better understand the meaning of r^2 by studying the figure at the right. The square $OXYZ$ has each side equal to the radius (r) of the circle; hence the area of the square $OXYZ$ is $r \times r$, or r^2. The area of the circle is $3\frac{1}{7}$ times as large as the area of this square; that is, the area of the circle equals $3\frac{1}{7}\, r^2$.

If a circle has a radius of 5 ft., its area is found by substituting 5 for r in the formula $A = 3\frac{1}{7}\, r^2$; this gives $A = 3\frac{1}{7} \times 5 \times 5$, or $78\frac{4}{7}$. Hence the area is $78\frac{4}{7}$ sq. ft.; if this result is rounded off to the nearest whole unit, it becomes 79 sq. ft.

Work these problems on a separate sheet of paper. Round off the results to the nearest whole unit. Let $\pi = 3\frac{1}{7}$:

(3) Mrs. Reed has two circular flower beds. One has a diameter of 16 ft. and the other has a diameter of 14 ft. Find the area of each bed. The formula $A = \pi r^2$ makes use of the radius; what is the radius of each flower bed?

(4) A calf was tied to a stake in a field by a rope 12 ft. long. What is the shape and the size (in square feet) of the area over which the calf could graze?

Find, to the nearest whole unit, the areas of these circles. Let $\pi = 3\frac{1}{7}$:

	a		*b*		*c*		*d*	
	Diameter	Area	Radius	Area	Diameter	Area	Radius	Area
(5)	28 ft.	------	6 yd.	------	70 in.	------	7 mi.	------
(6)	10 in.	------	11 yd.	------	44 ft.	------	2 yd.	------
(7)	20 ft.	------	13 ft.	------	32 yd.	------	8 ft.	------
(8)	18 yd.	------	15 ft.	------	21 in.	------	3 mi.	------

MIXED PRACTICE

Do any necessary work on a sheet of paper; then put the answers in the spaces:

1. The fraction $\frac{14}{25}$ changed to a per cent is _____%.

2. \$3.897 rewritten correct to the nearest cent is \$_____.

3. The formula for the area of a circle is _____.

4. The fraction that is equal to $87\frac{1}{2}\%$ is _____.

5. The fraction $\frac{75}{90}$ reduced to lowest terms is _____.

6. The average of 21, 28, 33, 35, 34, and 29 is _____.

7. A discount of 25% on \$284 equals \$_____.

8. $12\frac{1}{2}\%$ of \$160 is \$_____ less than $\frac{1}{4}$ of \$100.

9. The number that is 25% less than 72 is _____.

10. 2.7% of \$183.65, to the nearest cent, is \$_____.

11. 75% of a number is the same as _____ of it.

12. Joe is 5 ft. 6 in. tall. He is _____ in. tall.

13. Which mixed number is larger, $9\frac{3}{4}$ or $9\frac{11}{16}$? _____

14. The quotient of 18.31 divided by 2.4 is _____, to the nearest tenth.

15. When a man's salary increases from \$350 to \$420 a month, the per cent of increase is _____%.

16. The area of a trapezoid whose bases are 11 ft. and 15 ft. and whose height is 10 ft. is _____ sq. ft.

17. In the Baker School there are 320 pupils of whom 144 are boys. What per cent of the pupils are boys? _____%

18. In finding the area of a figure like a rectangle or a triangle, both of the dimensions must be expressed in the same unit of measure. If the base is 12 ft. and the height is 3 yd., first change _____ to _____. If the figure with the above dimensions is a rectangle, its area is _____. If it is a triangle, its area is _____.

IMPROVING BY PRACTICE

Improvement Test — Subtraction F1

Time 4 min.

	a	*b*	*c*	*d*
(1)	392272 242679	129867 67653	711111 547626	300008 54729
(2)	929378 233723	668435 73632	100106 31318	629896 169784
(3)	857537 169999	200000 50765	345693 193819	167448 87973

-------- RIGHT OUT OF 12 SCORE -------- LINE L

MIXED PRACTICE

(4) What part of a mile is 660 ft.? 5280 ft. = 1 mi. ------

(5) Find the average of 3263, 2807, 1540, and 2486. ------

(6) How many times is 3.5 contained in 777? ------

(7) Change $\frac{23}{35}$ to a decimal, correct to the nearest hundredth. ------

(8) What is 75% of $18.50, correct to the nearest cent? ------

(9) Find the average of $12\frac{3}{8}$, $9\frac{3}{4}$, $10\frac{15}{16}$, and $12\frac{7}{16}$. ------

(10) What per cent is equivalent to the fraction $\frac{5}{8}$? ------

(11) What is the area of a circle having a diameter of 14 in.? ------

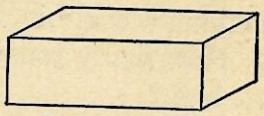
(12) How many times is $20\frac{1}{4}$ contained in 405? ------

(13) If 15% of a number is 126, find the number. ------

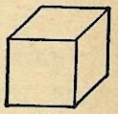
(14) Change $\frac{164}{201}$ to a per cent, correct to the nearest whole per cent. ------

-------- RIGHT OUT OF 11 SCORE -------- LINE K

A solid shaped like a box is called a *rectangular solid* because its 6 faces are all rectangles. If its 6 faces are all squares, it is a *cube.* A cube 1 in. long, 1 in. wide, and 1 in. high contains 1 *cubic inch.* What is a cubic foot? When you find the *volume* of a solid, you find the number of cubic units it contains.



Rectangular Solid



Cube

Volume of rectangular solid = length × width × height

This rule may be written as a formula, thus:

$$V = lwh$$

In finding the volume, first express all the dimensions in inches, or all in feet, or all in yards. Suppose the rectangular solid shown above has a length of 1 ft., a width of 8 in., and a height of 6 in. To find the volume, first change 1 ft. to 12 in., so that all the dimensions will be in inches. Then $V = 12 \times 8 \times 6$, or 576. The volume is 576 cu. in.

Work the following on a separate sheet of paper:

(1) A cubic yard is a cube that has each of its dimensions equal to 1 yd. Change 1 yd. to 3 ft. and show that 1 cu. yd. = 27 cu. ft.

(2) A cubic foot has each of its dimensions equal to 1 ft. Change 1 ft. to inches and show that 1 cu. ft. = 1728 cu. in.

(3) Mr. Smith is building a new house. The cellar, which has the shape of a rectangular solid, is to be 27 ft. long, 24 ft. wide, and $7\frac{1}{2}$ ft. deep. How many cubic feet does this cellar contain? What will it cost to dig this cellar at 90¢ a cubic yard?

(4) A box is 2 ft. long, $1\frac{1}{2}$ ft. wide, and 9 in. high. How many cubic feet does it contain? How many cubic inches does it contain?

These dimensions refer to rectangular solids. Put the answers in the spaces:

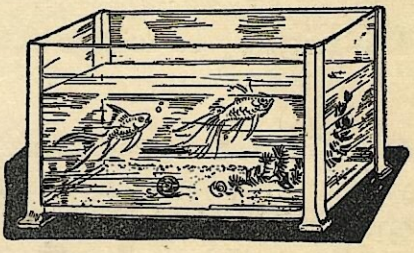
	Length	Width	Height	Volume
(5)	15 ft.	8 ft.	3 ft.	------
(6)	20 in.	9 in.	7 in.	------
(7)	10 ft.	4 ft.	------	80 cu. ft.
(8)	14 ft.	9 in.	------	21 cu. ft.

	Length	Width	Height	Volume
(9)	4 yd.	7 ft.	3 ft.	------
(10)	5 ft.	4 ft.	15 in.	------
(11)	6 yd.	------	3 yd.	90 cu. yd.
(12)	4 yd.	------	4 ft.	12 cu. yd.

Work the following problems on a separate sheet of paper:

1. In making a coal bin, it is necessary to provide 35 cu. ft. of space for each ton of hard coal. How many tons of hard coal can be put in a bin that is $7\frac{1}{2}$ ft. long, 7 ft. wide, and 6 ft. high?

2. A metal water tank has the shape of a rectangular solid; its length is 8 ft.; its width, 6 ft.; and its height, 5 ft. How many gallons of water will this tank hold when it is full? How many gallons will it hold when it is filled to a depth of 3 ft.? 1 cu. ft. $= 7\frac{1}{2}$ gal.



3. Mrs. Alexander has some goldfish in an aquarium that is $16\frac{1}{2}$ in. long, 8 in. wide, and 10 in. high. How many gallons of water will she have to put in this aquarium to fill it to a depth of 7 in.? Let 231 cu. in. = 1 gal.

4. In ex. 3, how many gallons of water will it take to fill the aquarium to a depth of 9 in.? to a depth of 6 in.? Find the answers to the nearest whole gallon.

5. Mr. Smith bought a can of oil for his car. The can was 7 in. wide, 11 in. long, and 15 in. high. How many gallons of oil did it contain? 231 cu. in. = 1 gal.

6. In building a new library a block of granite was used that had a base 4 ft. square and a height of 2 ft. If granite weighs 170 lb. per cubic foot, how many pounds did that block of granite weigh?

7. In ex. 6, does the block of granite weigh approximately 2 tons, $2\frac{1}{2}$ tons, $2\frac{3}{4}$ tons, or 3 tons? 1 ton = 2000 lb.

8. A rectangular bin is 8 ft. long, 6 ft. wide, and 5 ft. deep. How many bushels of shelled corn will this bin hold if 1 bu. $= 1\frac{1}{4}$ cu. ft.?

9. In ex. 8, find, to the nearest whole bushel, the number of bushels the bin will hold if filled only to a depth of 3 ft.

10. The formula for the volume of a rectangular solid, as given on page 103, is as follows:

$$V = \textit{length} \times \textit{width} \times \textit{height}$$

In the figure of the rectangular solid shown on page 103, it is seen that *length* × *width* gives the area of the *base* of the solid. Hence the formula for the volume of a rectangular solid may be written in another form, as follows:

$$V = \textit{area of base} \times \textit{height}$$

If the area of the base of a rectangular solid is 25 sq. ft. and the height is 7 ft., find the volume of the solid in cubic feet.

11. A swimming pool is 60 ft. long and 24 ft. wide and is filled to a depth of 5 ft. How many cubic feet of water does it contain? What did it cost to fill this pool if water costs 24¢ per 100 cu. ft.?

(1) Do Improvement Test F2 on Subtraction. See page 143.

The tin can shown here is a cylinder. Pears, peaches, peas, and string beans are usually canned in cylindrical cans. The volume of a cylinder may be found by this rule:

Volume of cylinder = area of base × height

Since the base of a cylinder is a circle, its area equals πr^2. Hence the rule above may be written as a formula, as follows:

$$V = \pi r^2 h$$

In this formula, r = radius of base of cylinder and h = height of cylinder.

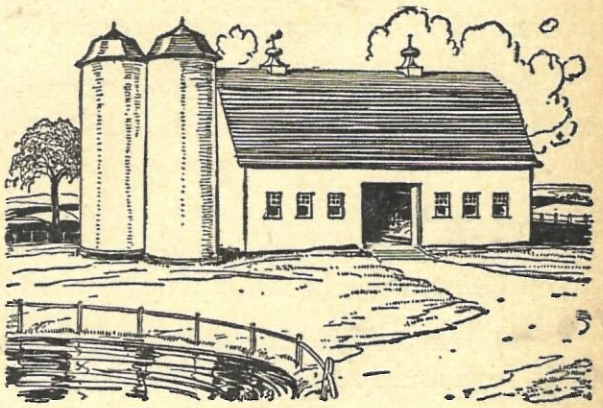
Suppose that a can is 5 in. high and that its base has a diameter of 4 in., or a radius of 2 in. To find the volume of the can, substitute these values in the formula, as follows:

$$V = 3\tfrac{1}{7} \times 2 \times 2 \times 5 = 62\tfrac{6}{7}$$

The can has a volume of $62\frac{6}{7}$ cu. in., or 63 cu. in. to the nearest cubic inch.

Work these problems on a separate sheet of paper. Let $\pi = 3\frac{1}{7}$:

(2) A gasoline truck has a cylindrical tank 4 ft. in diameter and 14 ft. long. How many gallons of gasoline will the tank hold? $7\frac{1}{2}$ gal. = 1 cu. ft. First find the radius of the tank.



(3) How many gallons of water will a cylindrical cistern hold if it measures 7 ft. in diameter and is filled with water to a depth of 8 ft.?

(4) Mr. Ford has built a new cow barn and at one end of it there are two silos of the same size. If the inside diameter of the cylindrical part of each silo is 10 ft. and the height is 28 ft., find the capacity of the two silos in cubic feet.

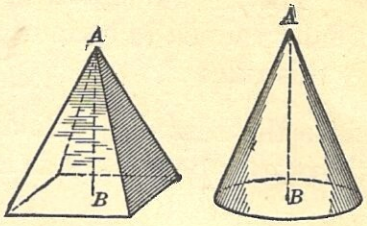
(5) If Mr. Ford fills one silo to a height of 21 ft. and the other to a height of 18 ft., how many cubic feet of silage, to the nearest cubic foot, has he for his cows?

Find, to the nearest cubic unit, the volumes of these cylinders. Let $\pi = 3\frac{1}{7}$:

	Radius of Base	Height	Volume		Radius of Base	Height	Volume
(6)	3 in.	8 in.	------	**(10)**	6 ft.	25 ft.	------
(7)	2 ft.	6 ft.	------	**(11)**	14 ft.	10 ft.	------
(8)	$1\frac{1}{2}$ in.	4 in.	------	**(12)**	$7\frac{1}{2}$ in.	12 in.	------
(9)	7 in.	45 in.	------	**(13)**	$8\frac{3}{4}$ in.	10 in.	------

PYRAMID AND CONE

The first solid at the right is a *pyramid.* The sides of a pyramid are all triangles. The base of a pyramid is usually a square, a triangle, or a rectangle. Line AB, which is perpendicular to the base, shows the *height* of the pyramid.



The second solid at the right is a *cone.* An ice-cream cone gets its name from its cone shape. The base of a cone is a circle. Line AB, which is perpendicular to the base, shows the *height* of the cone.

The rule for finding the volume of a pyramid is exactly the same as that for finding the volume of a cone. This rule is as follows:

$$Volume = \tfrac{1}{3} \times area\ of\ base \times height$$

Since the base of a cone is a circle, the area of the base is equal to πr^2. Hence the volume of a cone can also be found by this formula:

$$Volume\ of\ cone = \tfrac{1}{3}\pi r^2 h$$

In a cone, $h = 14$ in. and $r = 3$ in.; then $V = \frac{1}{3} \times 3\frac{1}{7} \times 3 \times 3 \times 14$, or 132, cu. in.

In these problems round off the results to the nearest whole unit:

(1) A sandstone monument, shaped like a pyramid, has a height of 18 ft. and a square base 4 ft. on each side. Find the volume of the monument. Find the weight of the monument if sandstone weighs 135 lb. per cubic foot.

(2) One of the Egyptian pyramids has a height of 300 ft. and a square base measuring 474 ft. on each side. If the pyramid is made of blocks of stone, each containing about 30 cu. ft., how many blocks are there in the pyramid?

(3) When a load of sand was sifted, it fell in the shape of a cone. If the pile was $3\frac{1}{2}$ ft. high and 10 ft. in diameter, how many cubic feet of sand did it contain?

Find the volumes of these pyramids or cones:

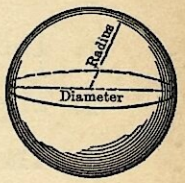
	Area of Base	Height	Volume
(4)	10 sq. in.	6 in.	------
(5)	15 sq. ft.	5 ft.	------
(6)	9 sq. in.	4 in.	------
(7)	20 sq. ft.	9 ft.	------

Find, to the nearest whole unit, the volumes of these cones. Let $\pi = 3\frac{1}{7}$:

	Radius of Base	Height	Volume
(8)	6 in.	15 in.	------
(9)	21 in.	30 in.	------
(10)	14 ft.	30 ft.	------
(11)	70 in.	150 in.	------

(1) Do Improvement Test F2 on Subtraction. See page 143.

The sphere is a very common solid. Tennis balls and baseballs are spheres. The *radius* of a sphere is a straight line from its center to any point on its surface; the *diameter* is twice the length of the radius. If a sphere is cut into halves, each half is a *hemisphere.*



The area of the outside surface of an entire sphere can be found by this formula, in which r is the radius:

$$\textit{Area of sphere} = 4\,\pi r^2$$

If a sphere has a radius of 5 in., the area of the entire sphere equals $4 \times 3\frac{1}{7} \times 5 \times 5$. How many square inches does the area contain?

The volume of an entire sphere can be found by the following formula, in which r^3 means $r \times r \times r$:

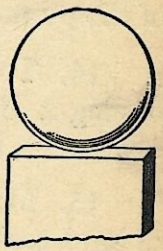
$$\textit{Volume of sphere} = \tfrac{4}{3}\,\pi r^3$$

If a sphere has a radius of 3 in., its volume equals $\frac{4}{3} \times 3\frac{1}{7} \times 3 \times 3 \times 3$. How many cubic inches does the volume contain?

Work the following problems. Let $\pi = 3\frac{1}{7}$:

(2) The earth is almost a perfect sphere. If its radius is about 4000 mi. long, what is the approximate number of square miles in the area of the entire surface of the earth? Round off the result to the nearest million square miles.

(3) The gilded dome on top of a state capitol is in the shape of a hemisphere with a diameter of 14 ft. What is the total area of the surface of this dome?



(4) A granite sphere 2 ft. in diameter rests on the top of a column. How much does this sphere weigh if 1 cu. ft. of granite weighs 168 lb.?

(5) How much would a granite sphere 4 ft. in diameter weigh? (See ex. 4.) A sphere 4 ft. in diameter is how many times as heavy as a sphere 2 ft. in diameter?

(6) A stack of hay is approximately a hemisphere 18 ft. in diameter. If 1 ton of hay occupies 500 cu. ft., about how many tons of hay are there in this stack?

Find the areas and volumes of spheres having these radii. Round off the results to the nearest whole unit. Let $\pi = 3\frac{1}{7}$:

	Radius	Area	Volume		Radius	Area	Volume
(7)	$3\frac{1}{2}$ ft.	------	------	**(10)**	14 in.	------	------
(8)	$10\frac{1}{2}$ ft.	------	------	**(11)**	10 in.	------	------
(9)	35 ft.	------	------	**(12)**	28 in.	------	------

DIAGNOSTIC TEST

To do ex. 1, look in the column at the right and find the formula for the area of a trapezoid; then write the number of this formula on the dotted line in ex. 1. Answer ex. 2 to 11 in like manner:

1. The area of a trapezoid _____

2. The volume of a sphere _____

3. The area of a triangle _____

4. The area of a sphere _____

5. The area of a circle _____

6. The area of a rectangle _____

7. The volume of a cylinder _____

8. The volume of a cone _____

9. The area of a parallelogram _____

10. The circumference of a circle _____

11. The volume of a rectangular solid _____

(1) $A = \frac{1}{2}bh$

(2) $V = \frac{1}{3}\pi r^2 h$

(3) $A = bh$

(4) $V = \pi r^2 h$

(5) $C = 2\pi r$

(6) $A = \frac{1}{2}h(a + b)$

(7) $A = \pi r^2$

(8) $A = lw$

(9) $V = lwh$

(10) $V = \frac{4}{3}\pi r^3$

(11) $A = 4\pi r^2$

12. In the formula $V = \frac{1}{3}Bh$, B represents the area of the base. This formula gives the volume of a _____ or a _____.

13. Suppose a pyramid has a triangular base whose area is 12 sq. in. If the height of the pyramid is 10 in., the volume of the pyramid is _____.

14. To find the number of cubic feet of gas it will take to fill a spherical balloon having a diameter of 14 ft., one must use this formula: _____. It will take _____ cu. ft. of gas, to the nearest cubic foot, to fill the balloon.

15. Mr. Adams is building a coal bin which must be large enough to hold 6 tons of hard coal. If he makes the bin 7 ft. long and 6 ft. wide, how many feet high should it be to hold 6 tons of coal? 1 ton hard coal = 35 cu. ft.

_____ RIGHT OUT OF 15

SCORE _____ LINE N

REMEDIAL EXERCISES

See page 138.

Ex. 1	p. 97	Ex. 6	p. 94	Ex. 11	p. 103
Ex. 2	p. 107	Ex. 7	p. 105	Ex. 12	p. 106
Ex. 3	p. 96	Ex. 8	p. 106	Ex. 13	p. 106
Ex. 4	p. 107	Ex. 9	p. 95	Ex. 14	p. 107
Ex. 5	p. 100	Ex. 10	p. 99	Ex. 15	p. 104

Improvement Test — Multiplication L1 Time 4 min.

	a	*b*
(1)	8327 432	3649 518
(2)	1946 256	6257 765

-------- RIGHT OUT OF 4 SCORE -------- LINE C

MIXED PRACTICE

(3) What is the product when you multiply 128 by 0? ------

(4) Which is greater, the sum of $\frac{7}{8}$ and $\frac{3}{4}$ or the sum of $\frac{11}{12}$ and $\frac{5}{6}$? ---------

(5) Find the average of 6.3, 11.2, 9.6, 15.6, 24.1, and 17.8. ------

(6) Find the product of 1285 and .0075. ------

(7) When the price of a dozen eggs increases from 56¢ to 70¢, what is the per cent of increase? ------

(8) Find the area of a triangle with a height of 17 ft. and a base of 10 ft. ---------

(9) If you take an arithmetic test containing 25 examples and get 3 examples wrong, your mark on this test should be ------%.

-------- RIGHT OUT OF 7 SCORE -------- LINE F

METRIC SYSTEM

Instead of measuring lengths and distances by the foot, the yard, and the mile, most countries in Continental Europe, Asia, and South America use a system of measurement called the *metric system*. The only prominent countries that do not employ the metric system for everyday use are the United States and Great Britain. The unit of length in the metric system is called the *meter*, which is a little longer than our yard. The approximate equivalents of the metric units of length are given below. The prefix *kilo* means 1000; thus, a kilometer is 1000 meters.

1 meter (m.) = 39.37 in., or 1.1 yd.
1 kilometer (km.) = 0.621 mi., or $\frac{3}{5}$ mi.

Work these problems on a separate sheet of paper. Round off the answers to the nearest whole unit:

1. The distance from Paris to Havre is 205 km. How many miles is that?

205 km. = 205 × 0.621 mi., or _____ mi. This becomes _____ mi. to the nearest mile.

2. A man in Brazil made a train trip of 1150 km. How many miles did he travel?

3. Mrs. Smith brought 10 m. of ribbon from South America. How many yards of ribbon is that?

4. A dealer brought 500 m. of cloth from South America to the United States. The cloth was sold at $3.50 a yard. How much in all was received for the cloth?

5. If an airplane flies at the rate of 300 mi. per hour, what is its speed in kilometers per hour? 1 mi. = 1.61 km.

6. In the Olympic Games one of the events is an 800-meter relay race. How many yards is that? Another event is the 5000-meter run. How many miles is that?

7. In Italy a man rode a bicycle 44,870 m. in 1 hr. How many miles is that?

8. Change these distances to miles, correct to the nearest mile:
55 km. = _____ mi.; 115 km. = _____ mi.; 283 km. = _____ mi.

9. Change these distances to kilometers, correct to the nearest kilometer:
85 mi. = _____ km.; 937 mi. = _____ km.; 450 mi. = _____ km.

10. In the Olympic Games in a recent year the best records in certain events were as follows: high jump, 2.12 m.; pole vault, 4.56 m.; shot put, 18.57 m.; running broad jump, 7.83 m.; javelin throw, 85.71 m. Express each of these distances in feet and inches, correct to the nearest whole inch.

1. Do Improvement Test L2 on Multiplication. See page 144.

The metric unit of weight is the *gram* while our unit of weight is the *pound*. In countries using the metric system, butter, meat, and similar articles are sold by the kilogram. The most common measures in metric weight and their equivalents are as follows:

1 kilogram (kg.) = 1000 grams (g.)
1 kilogram (kg.) = about 2.2 lb.
1 metric ton (t.) = 1000 kilograms

Work these problems on a separate sheet of paper:

2. Carlos is a South American boy. He says that he weighs 65 kg. How many pounds does Carlos weigh? About how many kilograms do you weigh?

3. Mr. Brown was allowed to carry 30 kg. of baggage when he traveled by first-class plane to Europe. How many pounds of baggage was he allowed?

4. Joe weighs 120 lb. If he stands on scales that weigh in kilograms, what should the scales record?

5. If you wish to buy the equivalent of a pound of candy when it is sold by the kilogram, about how many kilograms do you ask for?

6. The weight of a metric ton in pounds is about _____ lb.

7. A United States nickel (five-cent piece) weighs 5 grams. How many nickels will it take to weigh 1 kg.?

8. If you buy 4 kg. of butter in a country that uses the metric system, about how many pounds of butter is that?

Fill in the following spaces. 1 kg. = 2.2 lb.; 1 lb. = 0.45 kg.:

9. 4 lb. = ____ kg. 10 lb. = ____ kg. 5 kg. = ____ lb. 2 kg. = ____ lb.

10. 8 lb. = ____ kg. 16 lb. = ____ kg. 17 kg. = ____ lb. $9\frac{1}{2}$ kg. = ____ lb.

11. The metric unit for measuring liquids is the *liter* (*l.*), which equals *about* 1 quart in our measure. In metric countries, milk, gasoline, and other liquids are sold by the liter. Change each of the following to liters, using 1 qt. = 1 l.: 8 gal. = _____ l.; 3 qt. = _____ l.; $2\frac{1}{2}$ gal. = _____ l.; 3 pt. = _____ l.

12. If you are traveling by car in a country where the metric system is used and wish to buy the equivalent of 12 gal. of gasoline, how many liters do you ask for?

13. A plane taking off from Brazil for Africa carried 1900 l. of gasoline. How many gallons of gasoline did it carry? Use 1 l. = 1.06 liquid quart.

USING RATIOS TO COMPARE NUMBERS

Put the right numbers or words in the spaces:

1. We often compare two numbers like 8 and 4 by asking the question, "8 is how many times as large as 4?" To answer this question we divide 8 by 4, which gives 2. Hence 8 is 2 *times* as large as 4. We also say that the *ratio* of 8 to 4 is 2. How many times as large as 3 is 9? ------ What is the ratio of 9 to 3? ------

2. We may also compare 4 and 8 by asking the question, "4 is what part of 8?" This question is answered by dividing 4 by 8, which gives $\frac{1}{2}$. Hence 4 is $\frac{1}{2}$ of 8. We also say that the ratio of 4 to 8 is $\frac{1}{2}$. When two numbers are compared by dividing, the result is called their *ratio*. The ratio of 8 to 4 may be written as $\frac{8}{4}$, $8 \div 4$, or $8 : 4$. The sign : indicates *division* and is read *divided by* or *to*. 8 and 4 are the *terms* of the ratio. What part of 12 is 4? ------ What is the ratio of 4 to 12? ------

3. If we say that the chances that a certain thing will happen are "4 to 8" or "4 out of 8," we are using ratio. Since $\frac{4}{8} = \frac{1}{2}$ or $\frac{2}{4}$, we see that chances of "4 out of 8" are just the same as chances of "2 out of 4," or a chance of "1 out of 2."

A chance of 1 out of 3 is the same as ------ chances out of 12.

4. Tom and Joe were candidates for election as president of the Athletic Association. Tom won the election, the votes being 4 to 1 in his favor. What does "4 to 1" mean? If Tom received 172 votes, how many votes did Joe receive?

5. In the Ford School, 4 out of every 5 boys belong to the Athletic Association. In the Reed School, 8 out of every 10 boys belong to the Athletic Association. Is a record of 4 out of 5 better than, worse than, or the same as, a record of 8 out of 10? ------

6. If you wish to compare 3 in. with 1 ft., you must first change 1 ft. to 12 in. Then the ratio of 3 in. to 12 in. is $\frac{1}{4}$. Hence 3 in. $= \frac{1}{4}$ ft. What part of a pound is 4 oz.?

If the ratio of two measurements is to be found, the measurements must first be expressed in the same unit of measure.

In columns a and b write each ratio as a fraction in its simplest form. In column c put the right number in the space. Do the work mentally:

	a	*b*	*c*
7.	12 to 14 $\frac{6}{7}$	6 in. to 2 ft. -----	8 oz. = ----- lb.
8.	14 to 21 -----	3 oz. to 1 lb. -----	9 in. = ----- ft.
9.	25 to 10 -----	2 ft. to 3 yd. -----	2 ft. = ----- yd.
10.	24 to 30 -----	1 pt. to 2 qt. -----	1 in. = ----- ft.
11.	11 to 12 -----	6 in. to 1 yd. -----	3 qt. = ----- gal.
12.	80 to 20 -----	8 min. to 2 hr. -----	9 in. = ----- yd.

(1) Do Improvement Test L 3 on Multiplication. See page 144.

The ratios $\frac{30}{45}$ and $\frac{12}{18}$ are equal because each ratio reduces to $\frac{2}{3}$. An expression like $\frac{30}{45} = \frac{12}{18}$, which shows that two ratios are equal, is called a *proportion*. The numbers 30, 45, 12, and 18 are called the *terms* of the proportion. An important fact, which is true of any proportion, will now be pointed out. In the proportion at the right, multiply the numerator of each fraction by the denominator of the other fraction, as shown by the dotted lines. You find that these two products are equal. For example, $30 \times 18 = 540$; likewise, $45 \times 12 = 540$. These products are called cross-products. *In any proportion the cross-products are equal.*

$$\frac{30}{45} \times \frac{12}{18}$$

While there are four terms in every proportion, it often happens that only three of these terms are given. It then becomes necessary to find the value of the fourth term; this is done by making use of the fact that the cross-products are equal. For example, in the proportion at the right, the first term, a, is unknown but the other three terms are known. The value of a is found as follows: (1) Find the cross-products, which gives $18a = 540$. (2) Divide both sides of $18a = 540$ by 18, which gives $a = 30$; hence the unknown term is 30. (3) Check by substituting 30 for a in the original proportion.

$$\frac{a}{45} = \frac{12}{18}$$

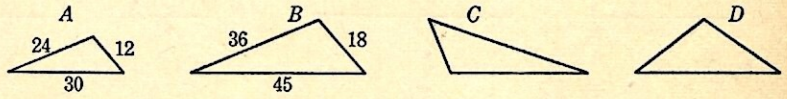
In ex. 2 show that the two ratios of each proportion are equal; also show that the cross-products are equal. In ex. 3 and 4 find the value of x in each proportion:

(2) $\frac{2}{3} = \frac{6}{9}$ $\quad \frac{5}{8} = \frac{15}{24}$ $\quad \frac{2}{12} = \frac{3}{18}$ $\quad \frac{4}{13} = \frac{16}{52}$ $\quad \frac{34}{38} = \frac{51}{57}$

(3) $\frac{x}{9} = \frac{10}{15}$ $\quad \frac{5}{6} = \frac{x}{18}$ $\quad \frac{x}{22} = \frac{4}{16}$ $\quad \frac{8}{x} = \frac{2}{7}$ $\quad \frac{9}{x} = \frac{12}{4}$

(4) $\frac{4}{9} = \frac{x}{36}$ $\quad \frac{12}{32} = \frac{6}{x}$ $\quad \frac{x}{26} = \frac{3}{18}$ $\quad \frac{45}{18} = \frac{15}{x}$ $\quad \frac{x}{45} = \frac{8}{9}$

Similar Triangles. Triangles having the same shape, such as A and B below, are called *similar triangles*. Triangles having different shapes, such as C and D, are not similar.



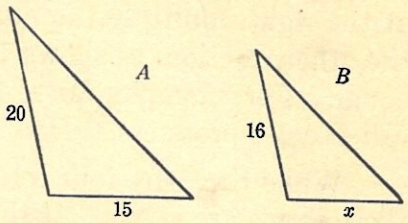
Study the similar triangles A and B. The longest side of triangle A equals 30; the longest side of triangle B equals 45. These longest sides form a pair of corresponding sides whose ratio is $\frac{30}{45}$ or $\frac{2}{3}$.

The shortest side of A is 12; the shortest side of B is 18. These shortest sides also make a pair of corresponding sides whose ratio is $\frac{12}{18}$ or $\frac{2}{3}$. Triangles A and B have still another pair of corresponding sides, namely, 24 and 36; the ratio of these sides is $\frac{24}{36}$ or $\frac{2}{3}$. The facts above may be summarized as follows: *In similar triangles the ratio of any pair of corresponding sides equals the ratio of any other pair.*

Since two similar triangles have three pairs of corresponding sides, each pair having the same ratio, it is easy to form proportions from these equal ratios. For example, the equal ratios given at the bottom of page 113 will make these proportions:

$\frac{30}{45} = \frac{12}{18}$ $\frac{30}{45} = \frac{24}{36}$ $\frac{12}{18} = \frac{24}{36}$

If you know that two triangles are similar but do not know the length of one side of one of the triangles, the missing side can often be found by using a proportion. For example, suppose you wish to find the base of triangle B which is unknown. Since the triangles are similar, you can write this proportion: $\frac{15}{x} = \frac{20}{16}$. By getting the cross-products you find $20x = 240$, hence $x = 12$.

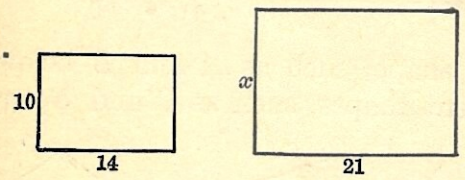


In triangles A and B it is seen that the lengths of the longest sides of these triangles are not given. Hence it is not possible to find the length of either of these sides by making a proportion from the sides that are given. It is possible, however, to find the length of one of these longest sides if the length of the other one is given. For example, if the longest side of A is 27, the longest side of B, which may be represented by y, may be found from this proportion: $\frac{27}{y} = \frac{20}{16}$. Find y.

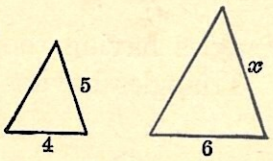
Rectangles having the same shape are also similar figures. In the similar rectangles shown below in ex. 1, the ratio of the heights, which is $\frac{x}{10}$, equals the ratio of the bases, which is $\frac{21}{14}$.

In each exercise below two similar figures are given. In ex. 1 to 5, find the length of side x. In ex. 6, find the lengths of sides x and y:

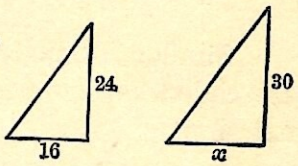
1.



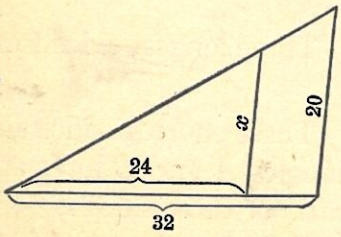
2.



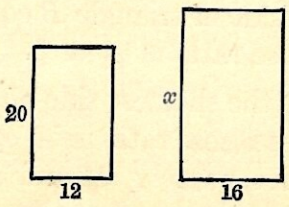
3.



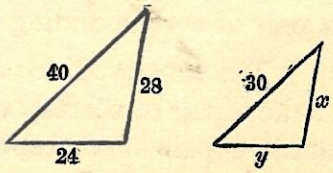
4.



5.



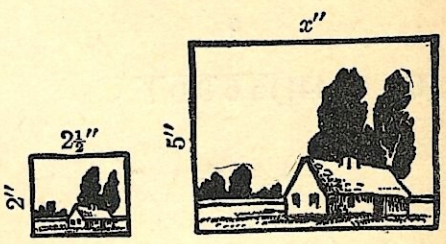
6.



NOTE. In ex. 4 note that the smaller triangle whose base is 24 overlaps the larger triangle whose base is 32. Since these triangles are similar you can form this proportion: $\frac{24}{32} = \frac{x}{20}$.

Write the answers in the spaces. Do the work on a separate sheet of paper:

1. Fred won a prize for a photograph 2 in. wide and $2\frac{1}{2}$ in. long that he took with his camera. This photograph is illustrated by the small picture at the right. If Fred has this photograph enlarged so that it will be 5 in. wide, how many inches long will it be? ------ When photographs are enlarged, the enlarged picture is similar to the original photograph. Let x = the length in inches of the enlarged picture.

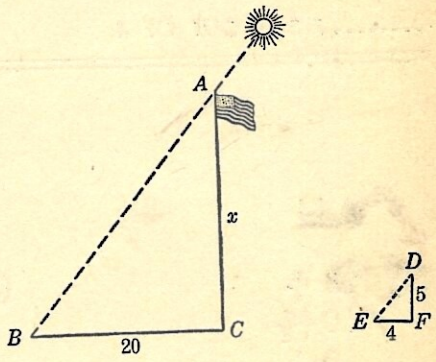


2. The official regulations for the dimensions of a United States flag state that the ratio of the length of the flag to its width should equal $\frac{19}{10}$. By these regulations, a flag that is 5 ft. wide should be ------ ft. long. Let x = the length in feet of the flag. Then $\frac{x}{5} = \frac{19}{10}$.

3. In the art class at school Elizabeth is copying a picture that is 8 in. long and 5 in. wide. If she makes her copy 14 in. long, how many inches wide must she make it? ------ If she makes a still larger copy 20 in. long, how many inches wide will that copy be? -------

4. The flagpole at the Clark School casts a shadow 20 ft. long at the same time of day that a fence post 5 ft. high casts a shadow 4 ft. long. How many feet high is the flagpole? ------

In the figure at the right AC is the flagpole. The dotted line AB, which shows the direction of the sun's rays, is drawn from the top of the pole to the end of its shadow. BC represents the shadow of the pole. DF is the fence post, EF is the shadow of the post, and DE shows the direction of the sun's rays. Triangles ABC and DEF are similar. In triangle ABC, AC is the side whose length is unknown; hence it is represented by x. Since the triangles are similar, $\frac{x}{5} = \frac{20}{4}$.



5. Yesterday noon a tree in the park cast a shadow 24 ft. long. At the same time Frank, who is $5\frac{1}{2}$ ft. tall, cast a shadow 4 ft. long. How many feet tall is the tree? ------

6. If a tree 35 ft. tall casts a shadow 75 ft. long at 5 P.M., how long is the shadow that a 56-foot flagpole casts at the same time of day? ------

7. In the art museum there is a picture 24 in. wide and 32 in. long which Mary wants to copy. If she makes her copy of the picture $7\frac{1}{2}$ in. wide, how many inches long must she make it? ------ If she makes her copy of the picture 12 in. long, how many inches wide must she make it? ------

IMPROVING BY PRACTICE

Improvement Test — Division V1

Time 4 min.

If any example has a remainder, write it as a fraction in the quotient:

	a	*b*
(1)	$441\overline{)56007}$	$812\overline{)510748}$
(2)	$629\overline{)222666}$	$576\overline{)396013}$

-------- RIGHT OUT OF 4

SCORE -------- LINE C

MIXED PRACTICE

Add

(3) $15\frac{3}{4}$
$10\frac{11}{12}$
$8\frac{1}{6}$
$14\frac{1}{8}$

Subtract

(4) $11\frac{1}{3}$
$8\frac{5}{6}$

Find the average

(5) \$6.80
7.93
9.15
4.06
5.25
8.39

Divide

(6) $18\frac{3}{4} \div 3\frac{1}{8}$

Multiply

(7) $20\frac{1}{4} \times 3\frac{1}{3}$

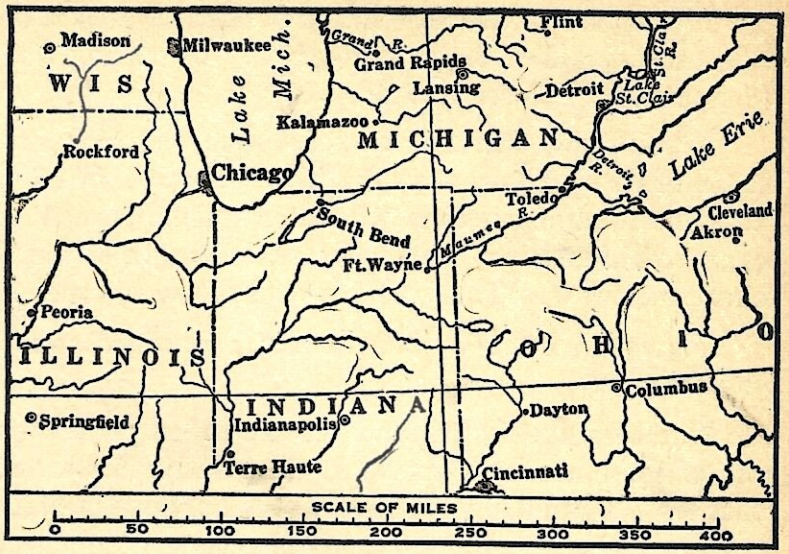
Find

(8) 2.8% of \$450

-------- RIGHT OUT OF 6

SCORE -------- LINE E

At the bottom of this map there is a scale for finding distances on this map. To find the airline distance between Akron and Toledo, lay the edge of a sheet of paper on the map so that it touches the dots representing these two cities. Make marks on the paper opposite these dots. Then place the paper on the scale, with one mark at the zero point, and read the distance opposite the other mark. The distance is 110 mi., to the nearest 10 mi.



Find, to the nearest 10 mi., the airline distance between the cities below:

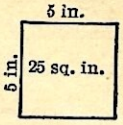
1. Flint to Peoria ------
2. Lansing to Dayton ------
3. Toledo to Madison ------
4. Madison to Peoria ------
5. Springfield to Dayton ------
6. Lansing to Ft. Wayne ------
7. Madison to Columbus ------
8. Indianapolis to Peoria ------
9. Columbus to Dayton ------
10. Indianapolis to Flint ------

11. The scale of a map is often given in the form of a ratio which is called the Representative Fraction (R.F.). For example, if the R.F. of a map is $\frac{1}{63360}$ or 1 : 63,360, it means that 1 in. on the map represents 63,360 in. on the ground. Since 63,360 in. = 1 mi., this means that 1 in. on the map represents 1 mi. on the ground; this is often called a scale of 1 mile to the inch. The numerator and denominator of the R.F. may also represent any other unit such as the foot, or the yard. Show that 63,360 in. equal 1 mile; use the fact that 5280 ft. = 1 mile.

12. Certain official maps of the United States are drawn on these scales: 1 : 500,000, 1 : 1,000,000 and 1 : 2,000,000. Show that 1 : 500,000 is a scale of *approximately* 8 mi. to the inch. Also show that 1 : 1,000,000 is a scale of *approximately* 16 mi. to the inch and that 1 : 2,000,000 is a scale of *approximately* 32 mi. to the inch.

13. On the map at the top of this page, show that 118 mi. on the scale equal about 1 in.; then show that this is approximately equivalent to an R.F. of 1 : 7,500,000.

If a square measures 5 in. on a side, its area is 5×5 sq. in., or 25 sq. in. 25 is called the *square* of 5. A short way of writing 5×5 is 5^2, which is read "5 square." You see that $5^2 = 25$.



If you know that the area of a square is 25 sq. in. and wish to find the length of its side, you must find the *square root* of 25. This means that you must find a number which, when multiplied by itself, gives 25. Since $5 \times 5 = 25$, the *square root* of 25 is 5. Hence 5 in. is the length of the side of the square. The sign for square root is $\sqrt{\ }$. Thus, $\sqrt{25}$ is read the *square root* of 25 and $\sqrt{25} = 5$.

The square root of a given number is the number which, if multiplied by itself, produces the given number.

Numbers like 49, 64, and 100 are called *perfect squares* because the square root of each of these numbers can be found exactly; for these particular numbers, the square roots are 7, 8, and 10 respectively. Numbers like 17, 24, and 50, however, are not perfect squares because the square roots of these numbers can be found only approximately. For example, the square root of 50 is 7.07 if the result is carried to two decimal places. This is an approximate result because 7.07^2 gives 49.9849 instead of 50. If the square root is found to three decimal places, it is 7.071, which is a still closer approximation since $7.071^2 = 49.999041$. In applied problems, in which the square root of 50 has to be used, the values 7.07 and 7.071 are sufficiently accurate for most purposes.

1. Find the square root of 697,225.

Step 1. Separate the number into groups of two figures each, starting with the decimal point; this gives 3 groups as follows: 69 72 25. There will be 3 figures in the square root since there are 3 groups of figures in the number.

Step 2. Find the largest square contained in the lefthand group, 69. The largest square is 64 and its square root is 8. Write 8 over 69. 8 is the first figure of the square root.

Step 3. Subtract 64 from 69 and bring down the next group of two figures. This gives 572 as a new dividend.

```
      8  3  5
   √69'72'25
      64
  160  5 72
  163  4 89
 ------------
 1660   83 25
 1665   83 25
 ------------
```

Step 4. Multiply 8, the part of the root *already found*, by 20, which gives 160. 160 is the *trial divisor*. Divide 572 by 160, which gives 3 as the second figure of the root. Write 3 above 72. Also add 3 to the trial divisor, 160, which gives 163 as the *complete divisor*. Multiply 163 by 3, which gives 489. Subtract 489 from 572, which gives 83. Since 83 is less than 163, 3 is correct as the second figure of the root.

Step 5. Bring down the next group, 25, thus forming a new dividend, 8325. Multiply 83, the part of the root already found, by 20, which gives 1660 as the new *trial divisor*. Divide 8325 by 1660, which gives 5 as the next figure of the root. Write 5 over 25. Also add 5 to the trial divisor, 1660, which gives 1665 as the *complete divisor*. Multiply 1665 by 5, which gives 8325. Subtract 8325 from 8325. Since there is no remainder, the work is completed.

Step 6. 835 is the exact square root of 697,225. Check by multiplying 835 by 835, which gives the original number, 697,225.

2. Find the square root of each number: 1369; 6724; 133,225; 374,544; 239,121; 609,961; 4356; 1849; 174,724; 58,081; 664,225; 2916; 961.

1. Find the approximate square root of 731; carry the result to 3 decimal places.

Step 1. Separate 731 into groups of two figures each, beginning at the decimal point; it is seen in this case that the lefthand group contains only one figure, 7.

Step 2. The largest square in 7 is 4. The square root of 4 is 2, which is the first figure of the root. Subtract 4 from 7 and bring down the next group of two figures. The new dividend is 331.

Step 3. Multiply 2, the part of the root already found, by 20, which gives 40 as the *trial divisor*. Divide 331 by 40, which gives 8 as the next *trial* figure of the root. Add 8 to 40 to form the *complete divisor*, 48. Multiply 48 by 8, which gives 384. Since 384 is larger than 331, 8 is too large as the second figure of the root. Try 7. Then the complete divisor is 47. $7 \times 47 = 329$. $331 - 329 = 2$. Since the remainder, 2, is less than 47, 7 is correct as the second figure of the root.

Step 4. Since there is a remainder of 2, 731 is not a perfect square. Hence groups of 2 zeros each should be annexed to 731 to the right of the decimal point. Bring down the first group of zeros, which gives 200 as a new dividend.

```
          2  7. 0  3  7
        √7'31.00'00'00
           4
      ------
40      3 31
47      3 29
      ------
540       2 00
540       0 00
      ------
5400      2 00 00
5403      1 62 09
      ------
54060     37 91 00
54067     37 84 69
      ------
              6 31
```

Step 5. The trial divisor is 20×27, or 540. Since 200 does not contain 540, the next figure of the root is 0. Add 0 to 540, which gives 540 as a complete divisor. Multiply 540 by 0, which gives 000. Subtracting 000 from 200 gives 200 as a remainder. Bring down the next group of zeros, which gives 20000 as a new dividend.

Step 6. The trial divisor is 20×270, or 5400; complete this step. Then continue the work until the square root has been found to 3 decimal places.

Step 7. The square root of 731, to 3 decimal places, is 27.037. The decimal point in the root is placed directly above the decimal point in the number.

To check the work, multiply 27.037 by itself. The product is 730.999369, which is a very close approximation to 731. If the root is desired only to the nearest hundredth, 27.037 is rounded off to 27.04.

To find the trial divisor, multiply the part of the root already found by 20. *To find the complete divisor, add the next figure of the root to the trial divisor.*

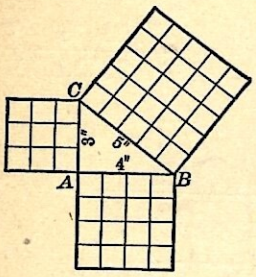
2. Find the approximate square root of 977.938.

Separate the number into groups of two figures each. To do this, begin at the decimal point and move to the left. Then begin again at the decimal point and move toward the right. This separates the number into these groups: 9 77 . 93 80. Note that a zero is annexed after 8 so that there will be two figures in the last group. The above grouping will give a square root to 2 decimal places.

Find the square root of each number. In ex. 4 to 6, carry the result to 3 decimal places and round off to the nearest hundredth:

	a	*b*	*c*	*d*	*e*
(3)	8836	5476	316,969	99,856	874,225
(4)	568	2935	276.513	25.874	240.085
(5)	385	7265	844.864	352.86	627.104
(6)	999	5941	962.211	4.0092	907.536

Square root is useful in finding the length of one side of a right triangle when the other sides are known.



In any right triangle, there is one right angle. The side opposite the right angle is called the *hypotenuse;* the other sides are called the *legs.*

Draw a right triangle, ABC, whose legs, CA and AB, measure 3 in. and 4 in. Measuring the hypotenuse, BC, you find that it is exactly 5 in. long. Now construct three squares having CA, AB, and BC as sides. Divide these squares into square inches. The square on leg CA contains 9 sq. in. The square on leg AB contains 16 sq. in. The square on the hypotenuse BC contains 25 sq. in., which is the sum of 9 sq. in. and 16 sq. in. Hence the square on BC equals the sum of the squares on the legs.

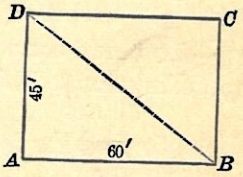
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This rule is called the *Pythagorean Theorem* and was discovered by Pythagoras, a Greek philosopher, about 540 B.C. The Pythagorean theorem may be expressed by the formula,

$$h^2 = a^2 + b^2$$

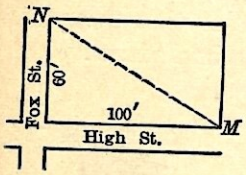
In the above formula h stands for the hypotenuse of the right triangle, and a and b stand for the other two sides.

1. Don can swim from one corner, B, of the swimming pool to the opposite corner, D. The pool measures 60 ft. by 45 ft. How many feet can Don swim?



In the right triangle ABD, let h represent the hypotenuse DB. Then $h^2 = 60^2 + 45^2$. Simplifying, you get $h^2 = 5625$. This means that the square on the hypotenuse equals 5625, hence the hypotenuse DB equals the square root of 5625. Find the square root of 5625, which represents the distance that Don can swim.

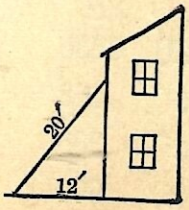
2. How many feet long must a stay wire be to reach from the top of a telephone pole 36 ft. high to a point on the ground 27 ft. from the foot of the pole? Draw a figure illustrating this problem.



3. An empty corner lot 100 ft. long and 60 ft. wide has a diagonal path, MN, leading across it. How many feet long, to the nearest whole foot, is this path?

4. In a skating race the boys were allowed to use sails containing not more than 18.5 sq. ft. If Tom makes a square sail, find to the nearest tenth of a foot, the greatest length which a side of Tom's sail could have.

5. A painter placed a ladder against the side of a house so that the distance from the foot of the ladder to the side of the house was 12 ft. If the ladder had a length of 20 ft., how many feet above the ground did the top of the ladder reach?



(1) Do Improvement Test V2 on Division. See page 144.

Add the following and check the work:

(2)

6182	$16\frac{2}{3}$	\$13.82	21	6.50
4937	$14\frac{1}{2}$	9.73	43	2.75
1818	$12\frac{3}{4}$	12.42	96	3.33
2407	$11\frac{7}{12}$	.86	82	7.25
3076	$5\frac{1}{6}$	19.72	75	1.67
1586		6.47	40	7.50
1927		15.50	67	4.75

Multiply or divide as indicated:

	a	*b*	*c*
(3)	$40 \div \frac{5}{8} =$	$7\frac{1}{2} \times 4\frac{2}{3} =$	$\frac{9}{16} \times \frac{2}{3} =$
(4)	$45 \times \frac{5}{6} =$	$8\frac{3}{4} \div 3\frac{1}{2} =$	$\frac{9}{10} \div \frac{3}{5} =$
(5)	$10 \times \frac{1}{5} =$	$2\frac{7}{10} \times 1\frac{2}{3} =$	$\frac{4}{5} \div 2 =$

Find these per cents, correct to the nearest cent:

(6)	28% of \$1550	$\frac{1}{4}$ of 1% of \$4824	$12\frac{1}{2}$% of \$72.64
(7)	0.5% of \$18.75	215% of \$93.55	$2\frac{3}{4}$% of \$68.65

-------- RIGHT OUT OF 20

SCORE -------- LINE T

A TABLE OF SQUARES AND SQUARE ROOTS

Engineers, machinists, and others who need frequently to find square roots make use of a table for this purpose; a part of such a table is given below.[1]

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1	1	1.000	21	441	4.583	41	1681	6.403	61	3721	7.810	81	6561	9.000
2	4	1.414	22	484	4.690	42	1764	6.481	62	3844	7.874	82	6724	9.055
3	9	1.732	23	529	4.796	43	1849	6.557	63	3969	7.937	83	6889	9.110
4	16	2.000	24	576	4.899	44	1936	6.633	64	4096	8.000	84	7056	9.165
5	25	2.236	25	625	5.000	45	2025	6.708	65	4225	8.062	85	7225	9.220
6	36	2.449	26	676	5.099	46	2116	6.782	66	4356	8.124	86	7396	9.274
7	49	2.646	27	729	5.196	47	2209	6.856	67	4489	8.185	87	7569	9.327
8	64	2.828	28	784	5.292	48	2304	6.928	68	4624	8.246	88	7744	9.381
9	81	3.000	29	841	5.385	49	2401	7.000	69	4761	8.307	89	7921	9.434
10	100	3.162	30	900	5.477	50	2500	7.071	70	4900	8.367	90	8100	9.487
11	121	3.317	31	961	5.568	51	2601	7.141	71	5041	8.426	91	8281	9.539
12	144	3.464	32	1024	5.657	52	2704	7.211	72	5184	8.485	92	8464	9.592
13	169	3.606	33	1089	5.745	53	2809	7.280	73	5329	8.544	93	8649	9.644
14	196	3.742	34	1156	5.831	54	2916	7.348	74	5476	8.602	94	8836	9.695
15	225	3.873	35	1225	5.916	55	3025	7.416	75	5625	8.660	95	9025	9.747
16	256	4.000	36	1296	6.000	56	3136	7.483	76	5776	8.718	96	9216	9.798
17	289	4.123	37	1369	6.083	57	3249	7.550	77	5929	8.775	97	9409	9.849
18	324	4.243	38	1444	6.164	58	3364	7.616	78	6084	8.832	98	9604	9.899
19	361	4.359	39	1521	6.245	59	3481	7.681	79	6241	8.888	99	9801	9.950
20	400	4.472	40	1600	6.325	60	3600	7.746	80	6400	8.944	100	10000	10.000

Using the Table. To find the square of 14, find 14 in the first column headed "No." Then, in the column headed "Square," just to the right of 14, you will find 196. 196 is the square of 14. In like manner, the square root of 14 is found to be 3.742, this value being given in the column headed "Square Root."

Using the Table Backward. The square roots of larger numbers that are perfect squares may be found by using the table backward. For example, to find the square root of 1681, find 1681 in one of the columns headed "Square." Then find the number to the left of 1681, which is 41; 41 is the square root of 1681.

The approximate square roots of larger numbers that are not perfect squares may also be found by using the table backward. For example, to find the square root of 1276 (which is not a perfect square and which is not in any column headed "Square"), pick out the two numbers in the table between which 1276 lies. These numbers are 1225 (whose square root is 35) and 1296 (whose square root is 36). Since 1276 lies between 1225 and 1296, its square root lies between 35 and 36. To find out whether the square root of 1276 is closer to 35 or to 36, find the difference between 1225 and 1276, which is 51, as shown at the right; then find the difference between 1276 and 1296, which is 20. Since 20 is a smaller difference than 51, this shows that 1276 is closer to 1296 than it is to 1225; hence the square root of 1276 is closer to 36 than it is to 35. Therefore the approximate square root of 1276 is 36, to the nearest whole number.

1225	
1276	51
1296	20

[1] One of the widely used tables of this kind is found in *Barlow's Tables*, which gives the squares and square roots, as well as the cubes and cube roots, of all whole numbers from 1 to 10,000.

(1) Do Improvement Test V3 on Division. See page 144.

By using the table, find both the square and the square root of each number:

(2)	15	27	33	61	84
(3)	45	20	57	77	65

By using the table backward, find the square root of each number:

(4)	4096	1369	784	6889	9409

By using the table backward find, to the nearest whole number, the approximate square root of each number:

(5)	1910	2726	4015	5127	596

(6) In a right triangle the lengths of the legs are 5 ft. and 6 ft. respectively. Find the length of the hypotenuse to the nearest tenth of a foot.

Find $5^2 + 6^2$. Then find the square root of the result, 61, by using the table. $\sqrt{61} = 7.810$, or 7.8 to the nearest tenth. The length of the hypotenuse is 7.8 ft.

In each exercise below, the given numbers represent the lengths of two of the three sides of a right triangle. Find the length of the missing side, to the nearest tenth of a foot. Use the table to get the square roots:

	Leg	Leg	Hypotenuse
(7)	3′	5′	------
(8)	2′	7′	------
(9)	3′	3′	------

	Leg	Leg	Hypotenuse
(10)	4′	----	8′
(11)	----	7′	10′
(12)	8′	3′	------

In each exercise below, find the length of the missing side of the right triangle. Use the table to find the squares. Use the table backward to find the square roots, correct to the nearest whole number:

	Leg	Leg	Hypotenuse
(13)	10′	24′	------
(14)	48′	55′	------
(15)	20′	21′	------
(16)	32′	----	40′
(17)	----	14′	50′

	Leg	Leg	Hypotenuse
(18)	31′	16′	------
(19)	35′	27′	------
(20)	14′	----	38′
(21)	----	23′	67′
(22)	50′	----	83′

READING BAR GRAPHS

SCHOOL ENROLLMENT

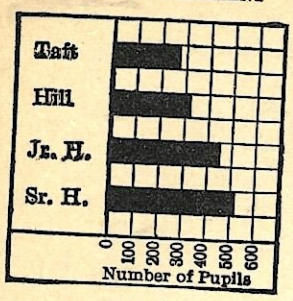


FIG. 1

RECORD OF PAPERS SOLD

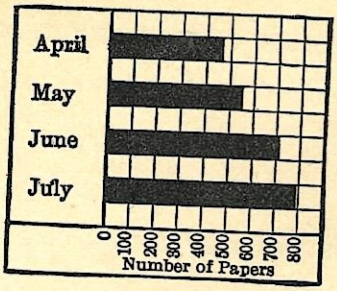


FIG. 2

NUMBER OF CARS SOLD

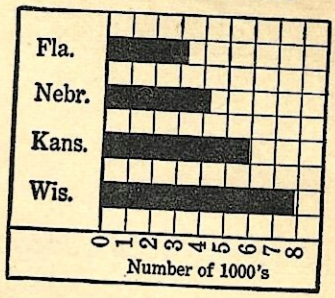


FIG. 3

Write the answers in the spaces:

1. The graph in Fig. 1 shows the number of pupils in each of 4 schools in Easton. Each square represents 100. The bar for the Taft School covers 2 squares and about $\frac{7}{10}$ of the next square; the $\frac{7}{10}$ of a square is *estimated* by the eye. Since $\frac{7}{10}$ of a square represents $\frac{7}{10}$ of 100, or 70, the entire bar for the Taft School represents *about* 270 pupils. The Taft School bar may really represent 269, 271, 273, or some other number close to 270; but in reading the bar, 270 is as close as one can come to it. When large numbers are represented by bar graphs, it is impossible to read the bars exactly. The bar for the Hill School covers 3 whole squares and about $\frac{2}{10}$ of the next square, so it represents about ------ pupils. The bar for the Junior High School represents about ------ pupils. The bar for the Senior High School represents about ------ pupils. The purpose of a graph like Fig. 1 is not to record the numbers precisely but to make possible a quick comparison of the enrollments in the 4 schools. The bars in a graph like Fig. 1 are usually arranged according to length, with the shortest bar at the top and the longest one at the bottom.

2. The number of newspapers that Joe sold in each of four months is shown in the bar graph in Fig. 2. The graph shows that Joe sold about ------ papers during April, about ------ papers during May, about ------ papers during June, and about ------ papers during July.

3. In a recent year in each of 4 states the Smith Motor Company sold the number of cars shown in the bar graph in Fig. 3. Each square represents 1000. In reading the Florida bar, you see that it covers 3 whole squares and about $\frac{3}{10}$ of the next square. Since each square in this graph represents 1000 cars, $\frac{3}{10}$ of a square represents ------ cars, and the entire Florida bar represents about ------ cars. The Smith Motor Company sold about ------ cars in Nebraska, about ------ cars in Kansas, and about ------ cars in Wisconsin.

NOTE TO TEACHER. In ex. 1 and 2, answers that differ by 10 from those given in the answers in the back of this book are also acceptable. Thus, if an answer is given as 720, an answer of 710 or 730 should also be accepted. In ex. 3 an answer may differ by 100 from the one given.

School Enrollment

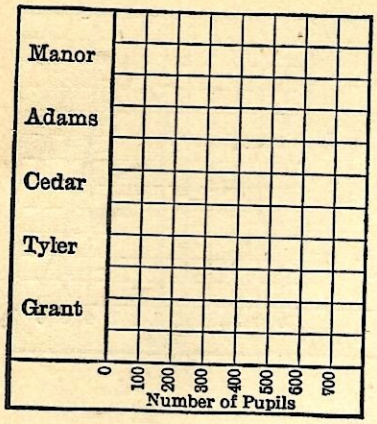


Fig. 1

Heights of Waterfalls

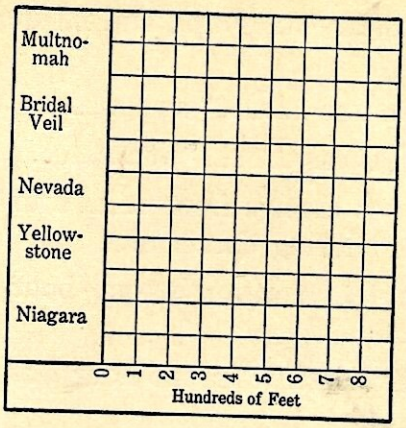


Fig. 2

Heights of Mountains

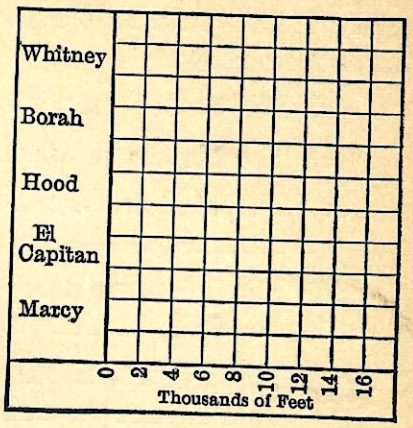


Fig. 3

1. The enrollment last year in five schools in Irvington was as follows: Manor, 327; Adams, 422; Cedar, 456; Tyler, 619; Grant, 783. In Fig. 1 above draw a bar graph to show these facts. Let 1 square = 100. To find the number of squares covered by each bar, divide each number by 100 and round off the result to the nearest tenth. Arrange the bars in order of length.

2. The heights of some famous waterfalls are: Multnomah (Oregon), 850 ft.; Bridal Veil (Yosemite), 620 ft.; Nevada (Yosemite), 594 ft.; Yellowstone (lower), 308 ft.; Niagara, 167 ft. Show these facts by a bar graph in Fig. 2.

3. The heights of five mountains are as follows: Whitney, 14,500 ft.; Borah Peak, 12,650 ft.; Mt. Hood, 11,250 ft.; El Capitan, 8700 ft.; Marcy, 5344 ft. In Fig. 3 draw a bar graph showing these facts, letting 1 square = 2000. Divide each number by 2000 and express the result correct to the nearest tenth to find the number of squares covered by each bar.

4. Draw a bar graph in Fig. 4 to show the populations of these cities: Waco, 84,706; Topeka, 78,791; Raleigh, 65,679; Elgin, 47,565. Let 1 square = 10,000. To find the number of squares covered by each bar, divide each number by 10,000 and round off the result to the nearest tenth.

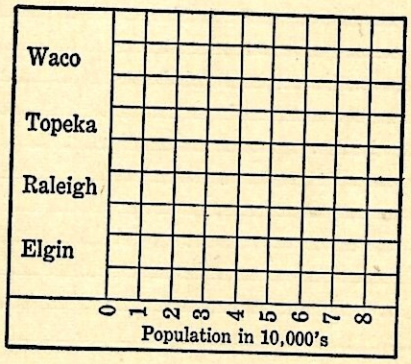


Fig. 4

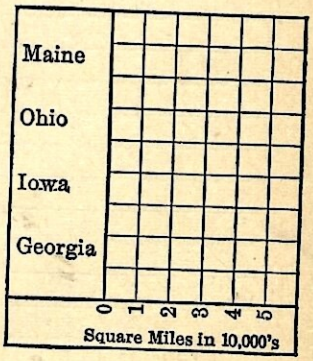
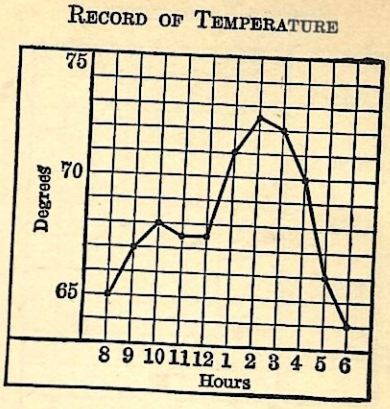


Fig. 5

5. The areas of four states in square miles are: Maine, 29,895; Ohio, 40,740; Iowa, 55,586; Georgia, 58,725. Draw a bar graph in Fig. 5 to show these facts. Let 1 square = 10,000.

LINE GRAPHS

1. The line graph at the right gives a temperature record from 8 A.M. to 6 P.M. Study the graph and fill in the spaces below:



(*a*) The lowest temperature was _____° at _____.

(*b*) The highest temperature was _____° at _____.

(*c*) The temperature did not change from _____ to _____ and was _____° during that hour.

(*d*) The greatest drop in temperature in one hour was from _____° to _____°.

(*e*) The greatest rise in temperature in one hour was from _____° to _____°.

(*f*) The temperature was _____° at 8.30, _____° at 9.30, _____° at 3.30, _____° at 4.15, _____° at 4.30, and _____° at 4.45.

(*g*) The temperature was 67° at _____ and also at about _____.

On the squared paper below, draw graphs to show these temperatures:

2.

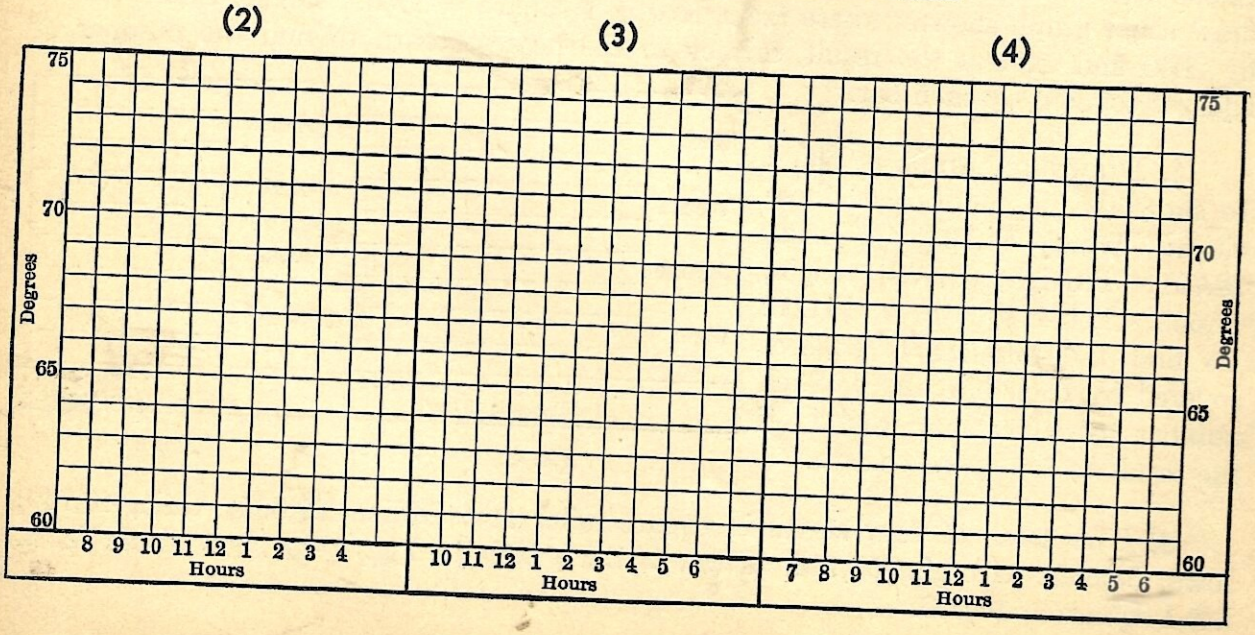
Hour	8	9	10	11	12	1	2	3	4
Degrees	63	65	69	71	73	74	74	72	71

3.

Hour	10	11	12	1	2	3	4	5	6
Degrees	71	73	75	74	74	72	71	69	67

4.

Hour	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	61	62	63	66	69	71	72	70	69	66	65	63



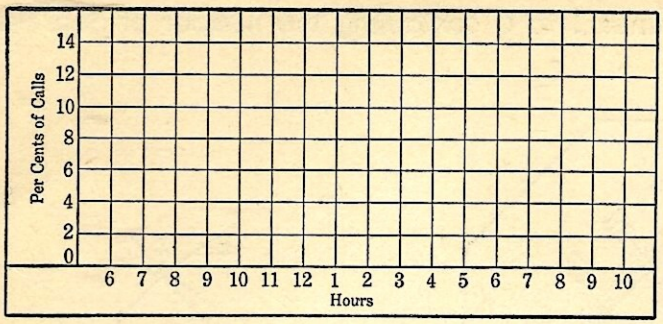


FIG. 1

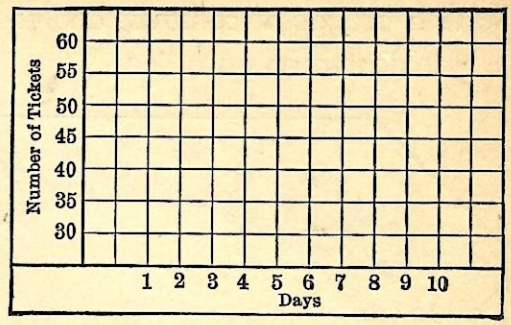


FIG. 2

1. Draw a line graph in Fig. 1 showing how telephone calls vary during a week day. The figures give the per cent of calls made each hour:

Hour	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10
Per Cent	$\frac{1}{2}$	2	6	10	9	8	7	8	7	7	$8\frac{1}{2}$	9	6	5	$3\frac{1}{2}$	$1\frac{1}{2}$	1

2. In Fig. 2 draw line graphs showing the number of tickets for the school play sold daily for 10 days by the boys and girls of the high school. Draw one graph for the boys, using a heavy line, and another for the girls, using a dotted line. Put both graphs in Fig. 2; one graph crosses the other:

Days	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
Boys	35	40	50	53	50	48	44	45	50	55
Girls	32	35	44	45	50	55	58	60	64	60

Study the graph that you made in Fig. 2 and write the correct words in the spaces:

(*a*) On the ------ day the boys and the girls each sold the same number of tickets.

(*b*) For the first four days the girls sold ------ tickets each day than the boys. Write *more* or *less* in the space.

(*c*) For the last five days the girls sold ------ tickets each day than the boys.

3. In Fig. 3 draw a line graph showing the growth in population in the United States:

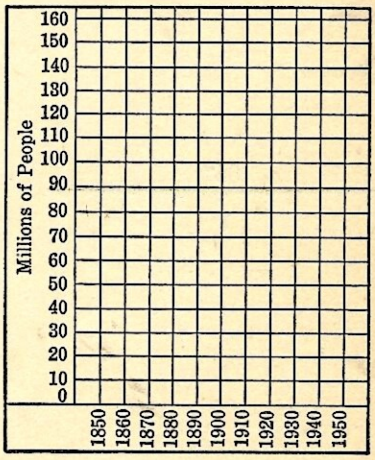
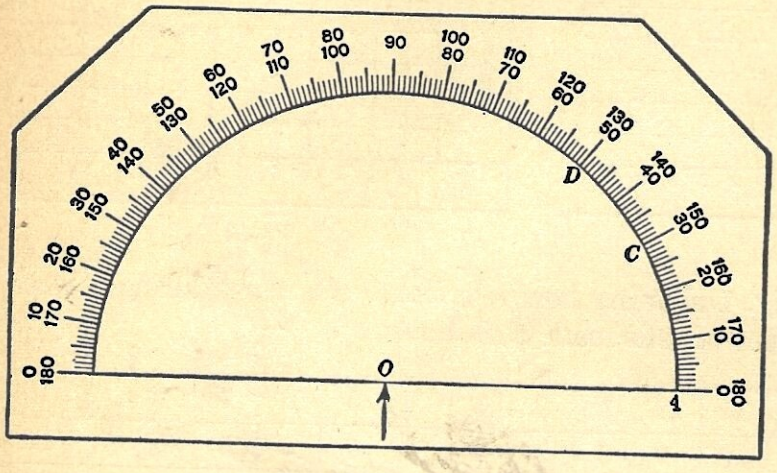


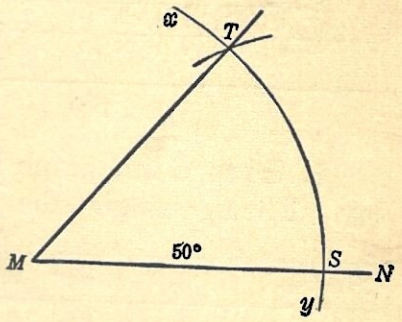
FIG. 3

Years	1850	1860	1870	1880	1890	1900	1910	1920	1930	1940	1950
Millions	23	31	39	50	63	76	92	106	123	132	151

USING A PROTRACTOR

With a protractor construct angles on lines 1 to 6 containing the number of degrees indicated on each line:





The center of the above protractor is the point below O at the tip of the arrowhead. The radius OA of this protractor is $1\frac{1}{2}$ in. If you do not have a protractor, the one above may be used to construct angles as follows:

Suppose you wish to construct an angle of 50° on line MN shown above at the right. With your compasses, using M as center and a radius of $1\frac{1}{2}$ in. (equal to the radius of the protractor), draw an arc xy cutting MN at S. Then place one point of the compasses on the circular edge of the protractor at the 0° mark (above A) and the other point on the circular edge at the 50° mark (near D). Using this distance as a radius and point S as center, draw an arc cutting arc xy at T. With your ruler draw a line connecting M and T. Then angle NMT is an angle of 50°.

65°
1

45°
2

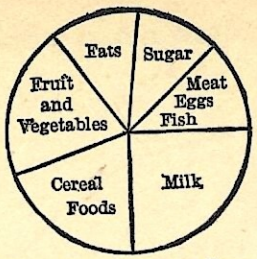
25°
3

75°
4

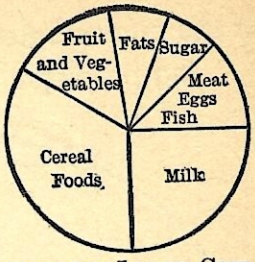
105°
5

32°
6

1. The graph marked Diet 1 gives a well-balanced diet where expense is not considered. The graph marked Diet 2 also gives a well-rounded and nutritious diet at a much lower cost. Each graph shows the proportion of the various kinds of foods that should go into the diet every day.



DIET 1 — HIGHER COST



DIET 2 — LOWER COST

Compare the two graphs; then put the right words or numbers in the spaces:

(*a*) Diet _____ requires less cereal foods than Diet _____.

(*b*) Diet _____ requires less fat than Diet _____.

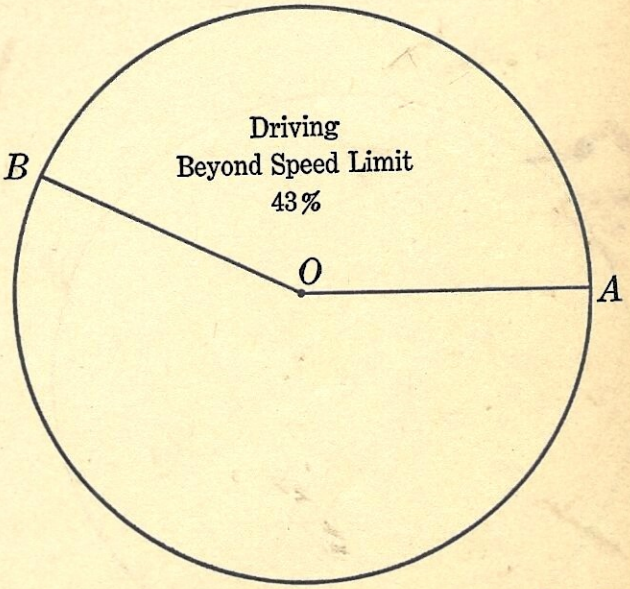
(*c*) The two diets have the same amount of _____ and also the same amount of _____.

(*d*) In Diet 1 the amount of fruit and vegetables is the same as the amount of _____.

2. In this country in a recent year the drivers of automobiles were responsible for about 31,800 deaths. 43% of these deaths were caused by driving beyond the speed limit; 27% were caused by reckless driving or driving on the wrong side of the road; 15%, by improper passing, wrong signaling, or driving off the roadway; 12%, by driving without having the right of way; and 3%, by other forms of wrong driving. Draw a circle graph showing these facts.

The whole circle, which equals 360°, represents *all* the deaths mentioned above. Since 43% of the deaths were caused by fast driving, use 43% of 360°, which equals 155°, to the nearest whole degree. To draw an angle of 155°, first draw the radius OA. Then with a protractor, draw an angle of 155° with OA as one of its sides, as you did on page 128. This will give angle AOB which represents the deaths due to fast driving.

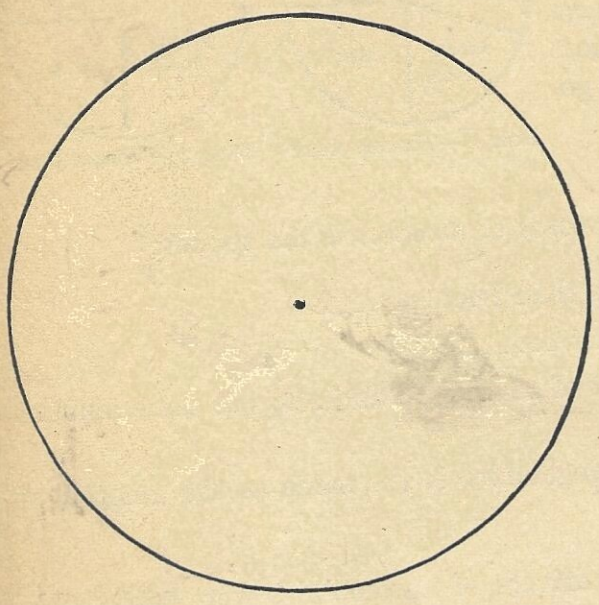
Complete the circle graph by drawing angles to represent the other four groups of deaths. In drawing the angle to show 27% of the deaths, use OB as one of its sides and draw the angle to the left of OB. The number of degrees in this angle will equal 27% of 360°. Find the size of each angle to the nearest whole degree.



3. Find the number of deaths due to each of the causes given in ex. 2 and write each number in the proper place on the graph in ex. 2.

DRAWING CIRCLE GRAPHS

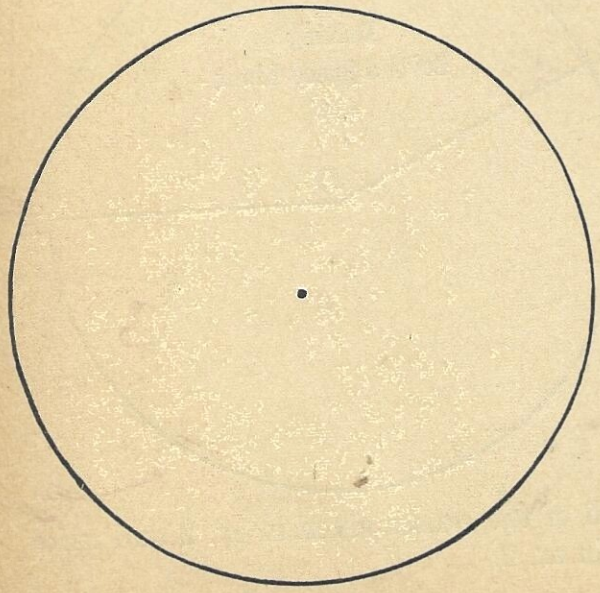
In the circles below, draw circle graphs showing the percentages given in the following problems:

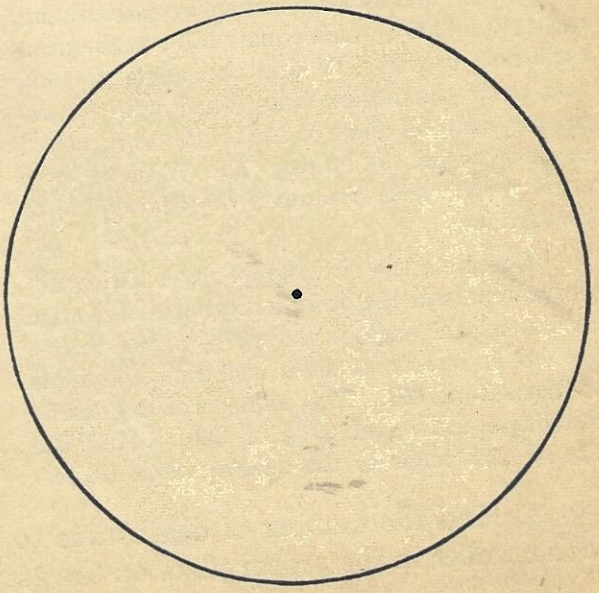


1. In a recent year 85% of the fatal automobile accidents in this country occurred on clear days, 3% on foggy days, and 12% on rainy and snowy days.

2. In the same year 81% of all these accidents occurred on dry roads, 15% on wet roads, and 4% on snowy and icy roads. We see that most accidents occur on dry roads when drivers are tempted to speed.

3. About 225,000 pedestrians are injured by automobiles each year because of carelessness. 45% of these accidents are caused by crossing streets against the signal or in the middle of the block, 15% by crossing at corners having no signal, 8% by coming from behind parked cars, 5% by children playing in the streets, and 27% by other careless actions on the part of pedestrians. After this graph has been drawn and the per cents have been inserted in their respective divisions, find the number of accidents due to each of the causes just mentioned and write each number in the proper place on the graph.





Write the answers in the spaces:

1. Add 62, 312, 483, and 76. Then multiply the sum by 12. ------

2. Divide 3921 by 43, carrying the result to 2 decimal places. ------

3. If 37 is one factor of 23,865, what is the other factor? ------

4. Write the formula for the area of a trapezoid. ----------

5. Change $\frac{7}{12}$ to a per cent, correct to the nearest tenth of 1%. ------

6. Add these mixed numbers: $7\frac{3}{4}$, $8\frac{1}{2}$, $2\frac{7}{8}$, $9\frac{1}{4}$, and $4\frac{5}{8}$. ------

7. Which is greater, 25% of \$525.00, or 30% of \$450.00, and how much greater? ----------

8. Find the square root of 334, correct to the nearest whole number. ------

9. Find the difference between \$341.26 and \$312.82. Then divide the difference by 12 and add \$7.23 to the result. ------

10. Find the product of 82.5 and .374; then add 1.073 to the result. Write the answer correct to the nearest hundredth. ------

11. What per cent discount is received when an article that is marked \$16.50 is sold at a sale for \$13.20? ------

12. At the County Fair the daily attendance for 6 days was as follows: 1325, 1286, 1314, 1075, 1105, 1407. Find the average daily attendance. ------

13. When the price of a gallon of gasoline increases from 28.5¢ to 31.5¢, what is the per cent of increase? Find the result correct to the nearest whole per cent. ------

14. Mary read that within a radius of 10 mi. of the town in which she lives there are 65 public schools. Approximately how many square miles, to the nearest whole square mile, does this cover? Let $\pi = 3\frac{1}{7}$. ------

15. Mr. Carter wishes to varnish two floors. One floor is 12 ft. by 15 ft. and the other is 15 ft. by 15 ft. 1 gal. of varnish will cover 600 sq. ft. If he puts two coats of varnish on each floor, how many gallons of varnish must he buy? The varnish comes only in gallon and half-gallon cans. ------

-------- RIGHT OUT OF 15

SCORE -------- LINE N

OPENING A BANK ACCOUNT

Suppose you own a bookstore and wish to open a bank account. Mr. Fred Beal, who is a depositor in the Taft National Bank, introduces you to the manager of that bank. You open the account as directed below.

1. Fill out the signature card below. Your address is 85 La Salle St., Chicago. Your telephone number is Central 8972.

2. Fill out the rest of the deposit slip below, using your name. Your deposit includes these items: bills, $315.00; specie, $29.25; a check for $16.25 on the Traders Bank of Peoria, the bank number being 70–178; a check for $123.29 on the Park Bank of Milwaukee; a check for $23.80 on a Chicago bank, 2–41.

3. Write your name on the pass book below. Your deposit is entered in it.

Signature Card. Write your name on the card the way that you expect to write it when you sign checks. If a man signs the card as Fred A. Lee, he should always sign his checks that way and not as F. A. Lee.

Deposit Slip. Many banks have numbers, which are printed on the check after the name of the bank. In listing checks on a deposit slip, use the number of the bank if it has one. If a bank has no number, list the check by the *name of the bank* if it is in your city, or by the *name of the city* if it is an out-of-town bank.

SIGNATURE CARD

TAFT NATIONAL BANK

Signature

Business

Address

Telephone

Introduced by

DEPOSIT SLIP

DEPOSITED IN THE
TAFT NATIONAL BANK

By

DATE May 6, 19—

BILLS		
SPECIE		
CHECKS		
LIST EACH CHECK SEPARATELY		
70-178	16	25
Milwaukee	123	29
TOTAL		

PASS BOOK

TAFT NATIONAL BANK
CHICAGO, ILL.

IN ACCOUNT WITH

NAME

May	6				507	59					

Sign your name to the checks below. Fill in the stubs and find the balances:

1. May 7. Sign check No. 1. Start the stub record with the deposit of $507.59 made on page 132. Carry the balance of $432.59 to stub No. 2.

2. May 9. No deposit. Write check No. 2 for $18.20 to pay Edward Hall his wages as a clerk in the bookstore.

3. May 9. No deposit. Write check No. 3 for $92.50 to the Star Pencil Company to pay for pencils purchased from that company.

STUB

No. 1 May 7, 19__
To John Martin
For rent

Balance Forward		
Deposited	507	59
Total	507	59
This Check	75	00
Balance Forward	432	59

CHECK

Chicago May 7, 19__ No. 1

TAFT NATIONAL BANK 2-37

Pay to the order of John Martin $75.00/100

Seventy-five and 00/100 Dollars

No. 2 19__
To
For

Balance Forward		
Deposited		
Total		
This Check		
Balance Forward		

Chicago 19__ No. 2

TAFT NATIONAL BANK 2-37

Pay to the order of $

Dollars

No. 3 19__
To
For

Balance Forward		
Deposited		
Total		
This Check		
Balance Forward		

Chicago 19__ No. 3

TAFT NATIONAL BANK 2-37

Pay to the order of $

Dollars

WRITING MORE CHECKS

Sign your name to all checks and deposit slips. Also fill in each line of the stub of each check:

1. Carry the last balance on stub No. 3 on page 133 to stub No. 4 below.

2. May 12. Fill out the deposit slip for these items: bills, $48.00; specie, $28.72; a check for $1.65 on a Boston bank, 5–183; a check for $3.85 on a Chicago bank, 2–89. Enter this deposit on the stub of check No. 4 below; also enter this deposit in your pass book on page 132.

3. May 12. Write check No. 4 for $145.27 to the Ajax Paper Co. for school pads. Find the last balance on stub No. 4 and enter this amount as the first balance on stub No. 5.

4. May 19. No deposit. Write check No. 5 for $95.23 to John L. Mason for books. Find the last balance on the stub of check No. 5.

DEPOSITED IN THE
TAFT NATIONAL BANK

By_____

DATE_____ 19___

BILLS		
SPECIE		
CHECKS LIST EACH CHECK SEPARATELY		
TOTAL		

No. 4 _____ 19___
To_____
For_____

Balance Forward		
Deposited		
Total		
This Check		
Balance Forward		

Chicago_____ 19____ No. 4

TAFT NATIONAL BANK 2-37

Pay to the order of_____ $_____

_____ Dollars

_____

No. 5 _____ 19___
To_____
For_____

Balance Forward		
Deposited		
Total		
This Check		
Balance Forward		

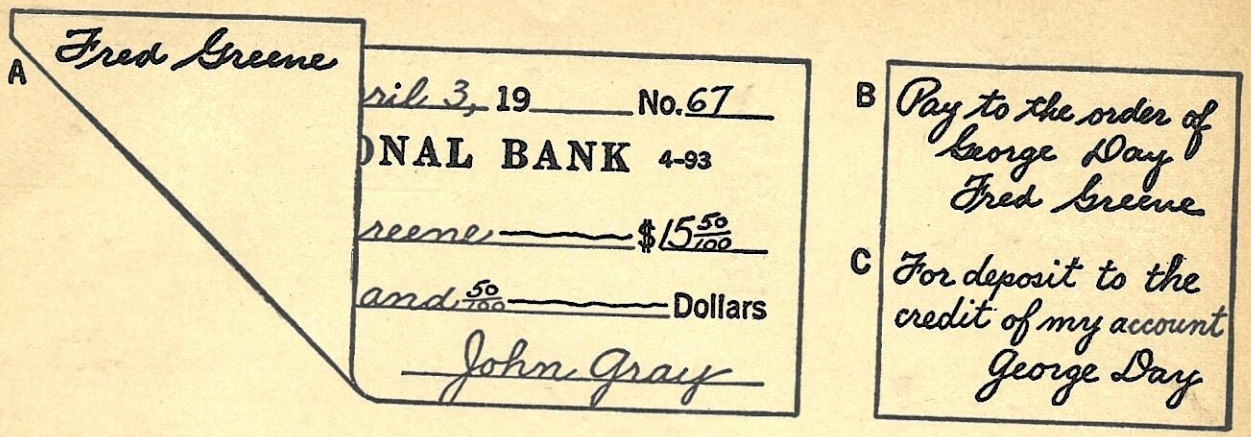
Chicago_____ 19____ No. 5

TAFT NATIONAL BANK 2-37

Pay to the order of_____ $_____

_____ Dollars

_____



Checks may be indorsed as follows: (1) *Indorsement in blank*, as shown at A, in which the name only is written; (2) *Indorsement in full*, as shown at B, which is a good way to indorse a check when the one who receives it gives it to another person; (3) *Indorsement for deposit only*, as shown at C, which is often used when checks are deposited in the bank to the credit of one's account. A check may have more than one indorsement, as shown at B and C; in this case the check was first given to Fred Greene, who gave it to George Day.

Always indorse a check across the back of the *left end* of the check, as shown above at A; the words "left end" refer to the left end of the face of the check.

Make the indorsements described in ex. 1 to 3 on the backs of the checks given at the bottom of the page:

1. Indorse in blank a check for $2.35, made out to you, that you have received in payment for a bill of goods.

2. Indorse, for deposit only, a check for $7.15 made out to you.

3. Indorse in full, so that it is payable to George Rose, a check for $25.00 for club dues, made out in your name. Up to May 1 you were the treasurer of the Golf Club; but Mr. Rose is now the treasurer, so you send the check to him.

1. **2.** **3.**

DIAGNOSTIC TEST

Add ex. a and b:

Multiply ex. c and d:

(1)

a	*b*	*c*	*d*
2.832	$6\frac{1}{4}$	2.75	25
4.916	$3\frac{3}{8}$	.035	$8\frac{3}{4}$
2.550	4		
1.625	$7\frac{1}{2}$		
8.933			
4.125			
6.025			

Write the answers in the spaces:

(2) Triangle: Base, 7′; height, 6′; area, _____.

(3) Trapezoid: Bases, 6′, 10′; height, 8′; area, _____.

(4) Circle: Radius, 3.5′; $\pi = 3\frac{1}{7}$; circumference, _____.

(5) Cone: Radius of base, 4.2′; height, 10′; volume, _____.

Divide. In ex. c and d, find the quotient correct to the nearest tenth:

(6)

a	*b*	*c*	*d*
$88\overline{)5456}$	$34\overline{)13513}$	$.62\overline{)15.932}$	$3.5\overline{)254.25}$

_____ RIGHT OUT OF 12

SCORE _____ LINE L

REMEDIAL EXERCISES

See page 138.

Ex. 1 *a* Ex. 1 *b* Ex. 1 *c*	p. 52, 1–3 p. 154, 1–7 p. 157, 6–13	Ex. 1 *d* Ex. 2–5 Ex. 6 *a*	p. 43, 4–8 pp. 96–99, 106 p. 153, 6–12	Ex. 6 *b* Ex. 6 *c* Ex. 6 *d*	p. 153, 1–5 p. 159, 9–14 p. 159, 9–14

Multiply	Divide	Change to a decimal
(1) $6\frac{2}{3} \times 4\frac{1}{2} =$	(2) $4\frac{1}{6} \div 1\frac{1}{4} =$	(3) $\frac{5}{16}$

Find the following. In ex. 5, get the root to the nearest tenth:

(4) $\sqrt{508369}$ (5) $\sqrt{70}$

Find the following. Where necessary, find results to the nearest cent:

	a	*b*	*c*
(6)	$2\frac{1}{2}\%$ of \$450	2.3% of \$9.75	$33\frac{1}{3}\%$ of \$8.62
(7)	115% of \$125	18 is _____% of 150	21 is 6% of _____

_____ RIGHT OUT OF 11 SCORE _____ LINE K

REMEDIAL EXERCISES *See page 138.*

Ex. 1	p. 156, 8–10	Ex. 5	p. 123	Ex. 7 *a*	p. 162, 1–4
Ex. 2	p. 156, 16–18	Ex. 6 *a*	p. 162, 5–8	Ex. 7 *b*	p. 163, 1–4
Ex. 3	p. 160, 1	Ex. 6 *b*	p. 162, 9–12	Ex. 7 *c*	p. 163, 9–12
Ex. 4	p. 118	Ex. 6 *c*	p. 161, 14–18		

HOW TO USE THE REMEDIAL EXERCISES

The remedial exercises given on pages 145–163 of this book afford material for meeting the individual needs of students. If a student does poorly on any set of exercises in this book, he may need additional practice on that work. In such cases the necessary practice material will be found on one of the Remedial Sheets. For example, if the student has difficulty in dividing decimals of the type given on page 67, he will find at the foot of that page a reference to suitable remedial exercises on page 159.

Answers should not be written on the Remedial Sheets (pp. 145–163), since these pages will be used over and over again. On some of the sheets the answers are to be written on folded paper and not on the sheet itself. For example, on Remedial Sheet 1 (p. 145) the student should place a sheet of paper with its upper edge immediately under the first row of examples, writing the answers along the edge of the paper. When the first row is finished, the answers should be folded under, the second row being worked along the folded edge, and so on. It will save time if the paper is folded in advance so as to form the creases. On some of the other Remedial Sheets the examples are to be copied and then worked.

HOW TO FIND CLASS SCORES FOR IMPROVEMENT TESTS

If the papers of a test are scored and put in a pile, with the scores arranged in order so that the highest are at the top and the lowest at the bottom, the median score, or class score, is found thus:

(1) If there is an odd number of papers, the score of the middle paper in the pile is the *median* score.

(2) If there is an even number of papers, there is no middle paper; instead there are two middle papers. In this case the median score is the average of the two middle scores.

If the two middle papers have scores of 9 and 7, the median score is 8. If they have scores of 9 and 8, the median score is the average of 9 and 8, or $8\frac{1}{2}$. This is called 9, however, since Improvement Tests are scored only in whole numbers.

A much quicker way of finding median scores is given below:

Suppose there are 15 students in the class. After the papers are scored, find out how many had a score of 10, then how many had a score of 9, etc. Then write on the board the number of students having each score, as shown at the right.

Score	*No. of Students*
10	3
9	6 ←
8	4
7	0
6	1
5	1
	15

Since there are 15 scores, the middle score is the 8th score, or the score of the 8th student. To find the median, count down in the column marked "No. of Students" until you reach the 8th student. In the first group there are 3 students; to these add 5 students from the next group in order to reach the 8th student. Since the 8th student belongs to the group of 6 students who received a mark of 9, the median score is 9.

In the above example the easy way to find that the 8th score is the middle score is to add 1 to the total number of students and divide by 2. Thus, $15 + 1 = 16$; $16 \div 2 = 8$. Hence the 8th score is the middle score. For a class of 14 students, you have $14 + 1 = 15$; $15 \div 2 = 7\frac{1}{2}$. The $7\frac{1}{2}$th score, which is halfway between the 7th and the 8th score, is the middle score.

Another illustration of the method of finding the median score is as follows: Suppose 8 students receive scores of 10, 6 students receive 9, 10 students receive 8, and 4 students receive 7. Since there are 28 students, we have $28 + 1 = 29$; $29 \div 2 = 14\frac{1}{2}$. Hence the $14\frac{1}{2}$th score, which is halfway between the 14th and 15th scores, is the median score. Counting the groups of students, you find that the 14th score is the last score in the group of 6 students whose mark was 9, while the 15th score is the first in the group of 10 students whose mark was 8. Hence the median score is $8\frac{1}{2}$, which is called 9.

As a third illustration, suppose that the class consists of 33 students and that 18 of them receive scores of 10, 11 receive scores of 9, and 4 receive scores of 8. The middle score in this case is the 17th, which falls in the group of 18 students whose score was 10. Hence the median score is 10.

Purpose of the Improvement Tests. This book includes a series of 60 Improvement Tests which are distributed at regular intervals throughout the book. The purpose of these Tests is to review and to develop further the computational skills with whole numbers that the students have learned previously, and to accomplish this task in a comparatively few minutes each week, so that ample time is left in each class period for the study of other topics.

Arrangement of the Tests. These Improvement Tests are arranged in sets, each set containing three tests *of the same kind and of equal difficulty*. For example, the first set of three tests, which are known as Tests A1, A2, and A3, is on addition (see pages 5, 7, and 10 of this book). It will be noted that Test A1 is given on page 5, while Tests A2 and A3 (which the students are directed to take on pages 7 and 10) actually appear on page 141. This work on addition is followed by a set of three tests on subtraction (Set D), which are scheduled on pages 11, 14, and 15. This is followed on pages 17, 19, and 21 by Set G on multiplication, and so on.

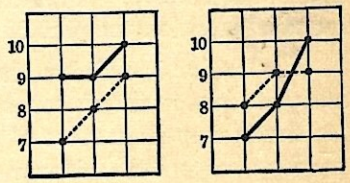
How to Give the Tests. The procedure in taking a test, such as Test A1 on page 5, is as follows:

(1) The students open the books to page 5, where Test A1 is given. At the signal "Go" the students begin to write the answers under the examples in the test. The time allowed for the entire test is 4 minutes. The test should be timed carefully, using a watch with a second hand.

(2) The teacher reads the right answers. Each student may check his own work, drawing a line through each example that is wrong. The student then counts the number of exercises that are right and scores the test on a scale of 10, making use of the Scoring Table that is printed on the inside of the back cover of this book.

(3) If a graphic record is desired, the student should record his score for Test A1 by placing a dot under A1 on the graph sheet on page 166 of this book. In a similar manner, when Test A2 is taken, the score should be recorded by a dot placed under A2, this dot being connected by a straight line with the dot under A1. Test A3, in turn, should be recorded in like manner.

(4) If it is desired, the *class score* can then be found. This enables each student to compare his score with that of the entire class. For the *class* score the teacher may use either the *average* score, which is found by taking the *average* of the scores of all the students, or the *median* score, which is described on page 138 of this book. The *median* score is preferable because it can be computed more quickly than the *average* score. The student may find it of interest to record the *class* score on the graph on page 166 along with his own score, as shown at the right. This makes it possible for the student to compare his scores with those of the class. On these graphs the heavy line represents the student's score and the dotted line, the class score. Let us study the two graphs at the right. In the first graph the student's score on the first test is 9 but the median score is only 7; on the second test the student's score is still 9 but the median score is now 8; on the third test the student's score is 10 while the median score is 9. It is evident that the student is ahead of the class, or median, score on each test. This means that the student is safely in the upper half of the class. In the second graph the student's score on the first test is 7 while the median score is 8; on the second test the student's score is 8 while the median score is 9; on the third test the student's score is 10 while the median score is 9. Hence the student is below the class, or median, score on the first two tests and above it on the third test; this record suggests that the student would profit by additional practice in order to make his performance more uniform. If desired, class scores may be recorded with a red pencil instead of by dotted lines.



When the students become familiar with the above procedure, the entire work of taking, scoring, and recording a test can be done in from 8 to 10 minutes.

(*Continued on page 140.*)

The procedure in taking Test A2 is like that outlined on page 139 for Test A1 except that the answers to Test A2 are not written in the book but on a piece of folded paper. Directions for using folded paper are given on page 138 under "How to Use the Remedial Exercises." The reason that answers for Test A2 are not written directly in the book is because this test is used again later on.

In taking those Improvement Tests on multiplication and division which are given on page 144, each test should first be copied on paper. The time assigned for each test on page 144 is the time actually allowed for working the test, *after the copying has been completed.*

Advantages of Graphic Record. Keeping a graphic record of the scores, as is done on page 166, enables the student quickly to tell whether his skill in a given operation is steadily improving. This graphic record also makes it possible for the student to compare his skill in one operation with that in another. For example, the student may find that he adds satisfactorily but that he multiplies and divides slowly and inaccurately. The graphic record quickly shows the student the operations upon which he needs the most practice.

How Often to Give the Tests. Only one Improvement Test is to be taken on a given day. These tests are so distributed throughout the book that they come about three times a week. When it is possible to do so, it is desirable to have the entire class take the same test at the same time since this makes it possible to compute the class score, which enables each student to compare his individual score with that of the class as a whole. This plan also maintains an *esprit de corps* and a class enthusiasm that is otherwise lacking. If students proceed at their own pace, obviously all of them will not be ready for the same test on the same day. In such cases, however, it is often desirable to have all students take the Improvement Tests at the same time, each student working on the test for which he is ready; such a plan is easily carried out since the time allowed for each test is 4 minutes.

If there are some students who need additional drill on the type of work found in a given set of three tests, the teacher may find it helpful to give these students a fourth test in that set; for example, after the class has completed Tests A1, A2, and A3, Test A2 may be repeated, thus making a fourth test in Set A. If, after a fourth test, a student's work is still unsatisfactory, his individual needs may be provided for through the remedial exercises.

In general, it will be found that the class makes a *median* score of 10 on the third test of each set. If the class has not made a median score of 10 on the third test, this is one indication that more drill is needed on examples of that type; hence in such cases it may be advisable to give a fourth test in the set, as described above. A failure to get a class median of 10 on the third test also indicates the necessity of remedial work on the part of those having the lower individual scores on the third test.

Attention is called to the fact that when a class obtains a median score of 10, it does not mean that every student in the class has a score of 10. If the scores of more than half the students of the class are 10, then the median score of the class will be 10. (For an illustration of such a case, see the last paragraph on page 138.) Hence certain students, who have not obtained individual scores of 10 on any of the tests of the preceding set, go forward to a new set of tests. This is purposely done in order to avoid discouragement on the part of the slower students, whose individual needs will be met by the exercises on the Remedial Sheets.

The Time Element. All the Improvement Tests are *timed*, in order to make it possible to compare the student's progress from day to day. If the student gets a score of 8 today and 9 tomorrow on the same kind of test, the score of 9 means little unless the time has been the same in each case. If the teacher prefers, the Improvement Tests may be given without the time element, though much interest and enthusiasm on the part of the students will be lost if the tests are not timed. In timing the tests it is important to use a watch *with a second hand* and to have all students stop *immediately* at the expiration of the given time.

Addition A2. Write results on folded paper: Time 4 min.

380	507	367	610	548	363
521	836	874	232	955	670
359	570	108	328	131	813
672	195	322	135	676	446
328	402	157	999	214	162
860	179	893	242	554	489
474	694	605	876	307	900
449	430	251	457	133	155

143

Addition A3. Write results on folded paper: Time 4 min.

105	382	200	425	863	715
781	325	194	541	339	131
347	478	212	127	234	340
879	762	376	769	680	564
244	500	849	154	452	878
518	686	371	802	700	967
753	199	545	796	579	615
151	408	369	465	323	429

Addition B2. Write results on folded paper: Time 4 min.

1805	2235	1713	4231	3633
1557	1669	1393	9047	7281
6239	4005	5144	2672	6534
2042	6917	6672	5090	7087
5523	1154	1826	1327	2935
8295	4811	3073	1937	4465
4937	5687	4380	8866	5709
1607	6564	5518	2426	3560

Addition B3. Write results on folded paper: Time 4 min.

5132	2004	1993	3436	4059
7352	3596	1844	6000	8415
2678	6903	7089	3543	7534
3957	4117	3451	2282	1160
7460	2875	3667	5696	1821
5848	5206	8789	7969	1579
9482	9998	6078	3506	9028
5009	4134	1200	2444	4668

Addition C2. Write results on folded paper: Time 4 min.

a	*b*	*c*	*d*	*e*
827	354	426	687	461
424	497	392	478	485
175	666	138	589	226
701	100	680	878	369
352	489	231	692	624
175	910	372	583	571
634	238	483	777	397
511	879	198	959	105
369	741	906	258	896
102	126	319	217	226

Subtraction D2. Write results on folded paper: Time 4 min.

16743 − 7046	70000 − 45133	42368 − 6432	76962 − 35315	14852 − 6666
42698 − 27958	16100 − 7599	45365 − 15571	15162 − 8723	22637 − 19999
18006 − 8254	68900 − 61270	23450 − 10862	54991 − 8703	64913 − 19979

Subtraction D3. Write results on folded paper: Time 4 min.

75002 − 41379	54949 − 35178	12923 − 4867	33823 − 14305	15100 − 7843
12189 − 4474	62317 − 33923	77624 − 8657	30000 − 1386	55228 − 32486
16544 − 7653	48077 − 13782	92462 − 79899	41313 − 15877	13810 − 8141

Subtraction E2. Write results on folded paper: Time 4 min.

661832 474688	728429 590844	271745 143094	622929 163894
400102 375508	731668 256865	500000 241287	273144 177777
381832 114735	959000 880736	857955 497508	539806 399999

Subtraction E3. Write results on folded paper: Time 4 min.

479631 192445	431681 231689	533529 187465	664000 125362
955369 276307	600000 325047	178579 123246	425122 259504
514489 225968	384584 106635	600302 216558	461397 185796

Subtraction F2. Write results on folded paper: Time 4 min.

425231 167901	493121 55679	500003 116428	134754 53258
400000 52169	622991 135886	173648 143958	527525 99999
119499 82935	748637 181679	700109 564214	117425 17426

In copying each test, spread the exercises over the paper, writing the first number (the multiplier) under the second number. The time given for each test is the time actually allowed to work the exercises **after** they have been copied:

G2 — 4 min.	G3 — 4 min.	H2 — 4 min.	H3 — 4 min.
a. 375 × 528	396 × 569	265 × 638	671 × 829
b. 269 × 417	427 × 872	306 × 452	346 × 713
c. 574 × 396	592 × 713	487 × 815	209 × 658
d. 219 × 837	648 × 837	529 × 947	235 × 298
e. 486 × 654	157 × 456	108 × 379	405 × 542
f. 138 × 192	318 × 437	743 × 763	879 × 964

J2 — 4 min.	J3 — 4 min.	K2 — 4 min.	K3 — 4 min.
a. 38 × 4826	53 × 9753	624 × 1728	549 × 2615
b. 72 × 8176	46 × 4892	503 × 3946	198 × 4739
c. 94 × 5376	74 × 1938	719 × 5280	207 × 3469
d. 85 × 3759	29 × 5324	285 × 8523	364 × 9450
e. 67 × 5049	87 × 2675		
f. 59 × 2481	68 × 8103		

R2 — 4 min.	R3 — 4 min.	S2 — 4 min.*	S3 — 4 min.*
a. 27,945 ÷ 81	23,214 ÷ 53	23,157 ÷ 83	17,018 ÷ 67
b. 23,958 ÷ 33	27,675 ÷ 45	20,325 ÷ 32	52,779 ÷ 73
c. 48,108 ÷ 57	57,767 ÷ 61	25,172 ÷ 58	11,346 ÷ 35
d. 18,675 ÷ 75	17,226 ÷ 29	12,201 ÷ 26	18,928 ÷ 28
e. 32,336 ÷ 86	12,350 ÷ 38	33,497 ÷ 41	49,007 ÷ 52
f. 38,472 ÷ 42	69,132 ÷ 84	30,832 ÷ 94	21,298 ÷ 46

T2 — 4 min.†	T3 — 4 min.†	U2 — 4 min.†	U3 — 4 min.†
a. 4772 ÷ 71	1960 ÷ 27	1783 ÷ 32	2884 ÷ 35
b. 1014 ÷ 25	2803 ÷ 91	4268 ÷ 65	1692 ÷ 29
c. 1104 ÷ 89	5046 ÷ 66	7329 ÷ 81	5327 ÷ 63
d. 2467 ÷ 44	5617 ÷ 59	4661 ÷ 77	2621 ÷ 46
e. 2417 ÷ 76	4107 ÷ 72	2550 ÷ 53	4082 ÷ 51
f. 1609 ÷ 37	3754 ÷ 43	3607 ÷ 48	6236 ÷ 87

L2 — 4 min.	L3 — 4 min.	V2 — 4 min.*	V3 — 4 min.*
a. 462 × 3476	375 × 6938	102,546 ÷ 422	79,464 ÷ 344
b. 943 × 2754	736 × 4218	74,957 ÷ 194	328,295 ÷ 517
c. 328 × 1593	184 × 9826	95,013 ÷ 621	129,807 ÷ 287
d. 491 × 7358	659 × 5674	380,224 ÷ 914	90,375 ÷ 723

* NOTE. In these tests, if there is a remainder, express it as a fraction in the quotient.
† NOTE. In these tests, find the quotient correct to the nearest tenth.

ADDITION AND MULTIPLICATION FACTS

Add or multiply, as the teacher directs. Use folded paper. (See page 138.)

(1)	3 × 6	1 × 5	4 × 7	1 × 1	6 × 1	0 × 7	1 × 2	6 × 6	2 × 2	1 × 0
(2)	3 × 9	7 × 4	5 × 3	8 × 6	5 × 0	5 × 9	3 × 4	6 × 3	2 × 6	0 × 3
(3)	7 × 9	9 × 4	3 × 2	4 × 4	0 × 8	1 × 7	5 × 6	4 × 0	7 × 2	6 × 8
(4)	3 × 1	4 × 5	8 × 2	2 × 0	0 × 9	4 × 1	7 × 5	6 × 4	1 × 3	9 × 1
(5)	6 × 7	3 × 0	5 × 4	4 × 8	4 × 6	2 × 4	4 × 9	5 × 1	1 × 8	0 × 6
(6)	5 × 7	7 × 7	0 × 2	3 × 3	6 × 5	2 × 8	5 × 5	9 × 0	5 × 8	4 × 2
(7)	2 × 1	7 × 8	2 × 3	0 × 1	8 × 3	6 × 9	1 × 4	7 × 0	9 × 3	3 × 8
(8)	6 × 2	9 × 2	8 × 8	7 × 6	6 × 0	3 × 5	9 × 6	2 × 7	8 × 0	8 × 1
(9)	8 × 5	5 × 2	9 × 8	7 × 1	1 × 9	0 × 0	2 × 9	7 × 3	9 × 5	8 × 4
(10)	9 × 9	4 × 3	3 × 7	0 × 4	9 × 7	8 × 7	1 × 6	2 × 5	8 × 9	0 × 5

NOTE TO TEACHER. This sheet may be used for reviewing either the addition or the multiplication facts. There are 100 addition facts and also 100 multiplication facts, all of which are given above. These addition and multiplication facts are alike so far as the number combinations are concerned, the only difference being that in one case the numbers are added while in the other they are multiplied.

REMEDIAL SHEET 2

ADDING BY ENDINGS

The teacher will give you a number like 7. You should then add 7 mentally to each of the numbers below, writing only the answers on folded paper. As you add, think like this: "13 and 7 are 20"; write 20. "25 and 7 are 32"; write 32 Do not write on this sheet:

(1)	13	25	32	18	14	31	19	66	10
(2)	22	16	24	37	68	33	59	15	21
(3)	11	34	47	12	36	39	23	35	38
(4)	27	58	20	17	89	83	96	62	70
(5)	92	86	43	48	81	45	30	72	84
(6)	78	85	69	57	61	56	63	40	29
(7)	67	76	64	87	49	50	65	71	88
(8)	53	90	46	74	55	97	42	54	28

NOTE TO TEACHER. After the pupils have added 7 to each number, as directed above, have the exercise done again on another folded paper, adding 3 instead of 7 to each number. Repeat until each number from 1 to 9 has been added. All the work above must be done mentally, writing only the answers on folded paper. This work is an important preparation for column addition, for carrying in multiplication, and for making change.

SUBTRACTION FACTS

Subtract, writing the answers on folded paper. Do not write on this sheet:

(1)	6 3	11 2	9 4	6 0	8 3	9 6	16 7	7 3	13 4	8 1
(2)	2 2	10 9	15 6	3 0	5 1	12 6	9 5	14 8	2 1	9 8
(3)	5 4	9 0	11 3	10 4	6 1	8 5	4 3	7 5	12 5	4 4
(4)	1 0	11 4	8 8	3 1	9 7	2 0	15 7	7 4	4 2	14 9
(5)	6 5	12 3	10 6	7 0	11 7	5 3	9 1	16 9	8 7	17 8
(6)	8 4	6 2	4 0	13 5	10 8	14 5	4 1	10 5	3 2	11 5
(7)	5 2	7 2	0 0	8 2	7 7	11 8	12 4	7 1	6 6	12 8
(8)	5 0	9 9	12 7	10 7	5 5	13 9	6 4	14 7	3 3	14 6
(9)	11 9	13 6	1 1	9 3	10 2	10 1	15 8	8 6	17 9	8 0
(10)	7 6	13 7	10 3	9 2	11 6	12 9	15 9	13 8	16 8	18 9

NOTE TO TEACHER. There are 100 subtraction facts, all of which are given above. This sheet may be used for written practice, using folded paper, and also for oral drill. Directions for using folded paper are given on page 138.

REMEDIAL SHEET 4

EVEN DIVISION FACTS

Divide, using folded paper. Do not write on this sheet:

(1)	$3\overline{)6}$	$4\overline{)8}$	$2\overline{)2}$	$5\overline{)20}$	$4\overline{)16}$	$2\overline{)10}$	$3\overline{)24}$
(2)	$5\overline{)0}$	$1\overline{)0}$	$1\overline{)4}$	$1\overline{)3}$	$3\overline{)12}$	$2\overline{)18}$	$5\overline{)10}$
(3)	$2\overline{)6}$	$3\overline{)0}$	$5\overline{)5}$	$3\overline{)15}$	$4\overline{)36}$	$1\overline{)2}$	$4\overline{)4}$
(4)	$1\overline{)5}$	$2\overline{)8}$	$2\overline{)0}$	$1\overline{)1}$	$5\overline{)25}$	$4\overline{)12}$	$5\overline{)15}$
(5)	$2\overline{)4}$	$3\overline{)9}$	$4\overline{)0}$	$3\overline{)3}$	$4\overline{)28}$	$3\overline{)18}$	$4\overline{)20}$
(6)	$6\overline{)0}$	$6\overline{)18}$	$6\overline{)36}$	$1\overline{)6}$	$2\overline{)12}$	$5\overline{)30}$	$4\overline{)24}$
(7)	$6\overline{)6}$	$6\overline{)24}$	$6\overline{)42}$	$6\overline{)30}$	$6\overline{)54}$	$6\overline{)12}$	$6\overline{)48}$
(8)	$7\overline{)7}$	$7\overline{)42}$	$7\overline{)28}$	$1\overline{)7}$	$2\overline{)14}$	$3\overline{)21}$	$5\overline{)35}$
(9)	$7\overline{)0}$	$7\overline{)21}$	$7\overline{)63}$	$7\overline{)35}$	$7\overline{)49}$	$7\overline{)14}$	$7\overline{)56}$
(10)	$8\overline{)0}$	$8\overline{)56}$	$8\overline{)32}$	$1\overline{)8}$	$2\overline{)16}$	$5\overline{)40}$	$4\overline{)32}$
(11)	$8\overline{)8}$	$8\overline{)64}$	$8\overline{)24}$	$8\overline{)72}$	$8\overline{)48}$	$8\overline{)16}$	$8\overline{)40}$
(12)	$9\overline{)9}$	$9\overline{)45}$	$9\overline{)63}$	$9\overline{)81}$	$1\overline{)9}$	$3\overline{)27}$	$5\overline{)45}$
(13)	$9\overline{)0}$	$9\overline{)72}$	$9\overline{)27}$	$9\overline{)63}$	$9\overline{)36}$	$9\overline{)18}$	$9\overline{)54}$

NOTE TO TEACHER. There are 90 division facts representing exact division, all of which are given above. Place the edge of the folded paper immediately under each row of exercises, writing only the answers along the edge of the paper.

UNEVEN DIVISION FACTS

Divide. Do the work mentally, writing the answers on folded paper. In the first example write the answer as 5 R1:

(1)	2)11	7)46	9)65	5)31	3)20	8)75	6)45
(2)	5)44	9)78	7)54	4)39	8)53	6)29	3)14
(3)	7)22	5)11	2)19	6)40	8)45	9)62	7)40
(4)	6)52	8)62	5)17	7)37	4)23	9)41	8)59
(5)	9)50	3)16	8)38	6)19	9)22	7)17	8)23
(6)	4)13	8)50	7)61	9)61	6)53	8)14	5)37
(7)	9)70	5)23	8)60	9)75	2)13	8)71	7)24
(8)	6)59	9)58	3)22	7)52	8)31	7)33	9)48
(9)	7)30	9)85	5)49	8)55	6)57	9)33	4)26
(10)	9)15	4)34	8)67	6)49	9)68	7)43	2)17
(11)	5)43	9)53	7)69	8)49	9)83	3)29	6)15
(12)	9)80	6)22	4)31	8)46	7)58	9)20	5)29
(13)	3)26	7)66	9)88	2)15	6)31	8)63	4)18
(14)	5)38	4)21	8)79	6)47	9)35	7)48	3)28
(15)	6)27	9)43	5)26	9)17	7)20	4)30	8)36

NOTE TO TEACHER. There are 360 uneven division facts of which a representative selection is given above. The more difficult facts are those with divisors of 6, 7, 8, and 9, hence these facts require more practice than those with divisors of 2, 3, 4, and 5.

REMEDIAL SHEET 6

ADDITION OR SUBTRACTION WITH CARRYING OR BORROWING

Add or subtract, as the teacher directs. Write the results on folded paper:

	a	*b*	*c*	*d*	*e*
(1)	4601 3587	71147 41276	45098 15470	616916 378966	406346 252969
(2)	3639 2642	63144 47864	37703 19168	962707 573805	836390 289746
(3)	7491 5951	77426 73928	94569 75874	812202 339536	409238 191458
(4)	6228 1833	81833 44179	66293 27413	951493 687925	607315 552666
(5)	6369 2709	71364 34897	50818 33399	464857 285848	732143 496846
(6)	5084 1256	62339 15037	72346 53487	320671 271282	822266 185349
(7)	8000 4468	25000 10073	60027 33500	200000 189954	476000 190006
(8)	4000 2123	70010 23791	80021 25906	738074 300095	530000 269082

NOTE TO TEACHER. This page may be used for practice either on addition or subtraction with carrying or borrowing. Rows 7 and 8 provide for important zero difficulties.

COLUMN ADDITION

Add. Write the results on folded paper:

	a	*b*	*c*	*d*	*e*	*f*
(1)	543	949	565	482	393	688
	775	435	343	209	538	210
	473	676	918	466	167	691
	156	798	607	928	508	452
	738	964	856	359	712	416
	387	882	774	245	450	131
	950	107	263	722	392	409
(2)	667	463	535	283	566	669
	374	284	151	319	339	977
	753	894	266	193	878	151
	839	548	995	825	489	347
	382	724	788	761	604	388
	562	978	725	708	521	855
	170	160	927	514	510	390
	927	405	701	941	366	209
(3)	641	316	104	152	482	893
	239	463	675	737	167	309
	274	300	587	463	844	771
	796	917	231	775	610	447
	449	559	679	681	345	565
	236	212	640	908	158	486
	108	690	803	376	599	659
	782	581	322	129	862	216
(4)	274	736	828	683	668	594
	947	388	535	624	646	974
	254	799	466	554	279	787
	909	475	874	877	958	599
	372	374	682	953	684	625
	391	285	776	889	992	634
	221	422	972	382	577	934
	566	362	195	207	136	167

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

Multiplication. Multiply and check. First copy the exercises on a sheet of paper. In copying, write the first number (the multiplier) in each example under the second number:

	a	*b*	*c*	*d*
(1)	49 × 239	78 × 216	629 × 378	793 × 126
(2)	53 × 523	45 × 627	547 × 586	349 × 506
(3)	28 × 536	71 × 679	835 × 349	957 × 278
(4)	56 × 681	93 × 230	289 × 345	207 × 694
(5)	17 × 718	60 × 581	817 × 734	589 × 329
(6)	78 × 429	34 × 781	456 × 619	517 × 308
(7)	827 × 467	319 × 743	164 × 389	67 × 1963
(8)	246 × 934	968 × 812	875 × 184	28 × 2567
(9)	939 × 416	147 × 416	398 × 502	93 × 3560
(10)	450 × 792	650 × 309	208 × 492	40 × 6079
(11)	607 × 123	104 × 904	578 × 802	14 × 8006
(12)	370 × 605	508 × 405	328 × 870	35 × 5601

Division. Divisors Ending in 1 to 5. Copy and divide. Check by multiplying:

	a	*b*	*c*	*d*	*e*
(13)	1071 ÷ 51	1793 ÷ 35	38,656 ÷ 64	19,800 ÷ 55	$261.12 ÷ 64
(14)	6699 ÷ 93	6552 ÷ 21	17,675 ÷ 35	13,095 ÷ 62	$186.75 ÷ 45
(15)	1778 ÷ 34	3413 ÷ 63	20,944 ÷ 34	17,998 ÷ 35	$109.72 ÷ 52
(16)	8904 ÷ 84	3840 ÷ 32	27,979 ÷ 54	12,915 ÷ 21	$197.28 ÷ 63
(17)	1054 ÷ 62	8910 ÷ 81	13,904 ÷ 71	17,350 ÷ 41	$176.80 ÷ 34
(18)	2798 ÷ 45	1584 ÷ 72	21,584 ÷ 93	11,713 ÷ 53	$308.20 ÷ 72
(19)	2025 ÷ 75	7575 ÷ 25	13,490 ÷ 61	37,758 ÷ 93	$314.96 ÷ 62
(20)	4796 ÷ 44	8060 ÷ 31	10,400 ÷ 65	25,959 ÷ 51	$468.77 ÷ 65
(21)	2279 ÷ 43	7325 ÷ 62	32,277 ÷ 53	16,863 ÷ 73	$140.39 ÷ 42
(22)	3345 ÷ 53	3998 ÷ 85	41,553 ÷ 81	17,820 ÷ 22	$116.61 ÷ 23
(23)	6450 ÷ 43	1394 ÷ 41	27,720 ÷ 63	29,400 ÷ 84	$179.98 ÷ 35

REMEDIAL SHEET 9

LONG DIVISION

Correcting Quotient Figures. Divisors Ending in 1 to 5. Divide and check:

	a	*b*	*c*	*d*	*e*
(1)	6324 ÷ 93	8084 ÷ 43	16,985 ÷ 43	48,120 ÷ 61	$262.15 ÷ 35
(2)	1034 ÷ 22	3276 ÷ 84	73,950 ÷ 85	20,350 ÷ 55	$497.04 ÷ 63
(3)	3078 ÷ 54	8211 ÷ 23	40,338 ÷ 83	12,050 ÷ 31	$152.64 ÷ 53
(4)	6052 ÷ 34	8568 ÷ 24	30,250 ÷ 55	30,000 ÷ 52	$351.33 ÷ 51
(5)	2418 ÷ 31	9125 ÷ 25	21,352 ÷ 34	12,044 ÷ 44	$329.24 ÷ 43

Divisors Ending in 6 to 9. Divide and check:

(6)	3788 ÷ 59	3041 ÷ 86	30,961 ÷ 57	46,782 ÷ 89	15,003 ÷ 27
(7)	9465 ÷ 76	2896 ÷ 67	11,901 ÷ 46	25,695 ÷ 59	42,359 ÷ 98
(8)	9313 ÷ 39	3661 ÷ 38	18,070 ÷ 38	56,859 ÷ 77	31,379 ÷ 66
(9)	1524 ÷ 47	4874 ÷ 77	49,658 ÷ 56	14,679 ÷ 69	46,772 ÷ 49
(10)	9926 ÷ 29	8227 ÷ 46	28,003 ÷ 36	35,996 ÷ 47	18,464 ÷ 56
(11)	5896 ÷ 47	2774 ÷ 88	14,011 ÷ 19	54,993 ÷ 87	49,846 ÷ 76
(12)	4422 ÷ 56	1654 ÷ 37	35,154 ÷ 69	24,374 ÷ 37	16,504 ÷ 38

Correcting Quotient Figures. Divisors Ending in 6 to 9. Divide and check:

(13)	1779 ÷ 57	5776 ÷ 19	19,458 ÷ 47	35,796 ÷ 38	15,947 ÷ 37
(14)	6263 ÷ 88	2412 ÷ 67	15,281 ÷ 37	53,694 ÷ 57	26,096 ÷ 28
(15)	3128 ÷ 46	7992 ÷ 36	18,984 ÷ 28	19,152 ÷ 36	63,987 ÷ 77
(16)	6403 ÷ 19	4256 ÷ 56	24,932 ÷ 46	20,938 ÷ 29	53,368 ÷ 56
(17)	2992 ÷ 49	8778 ÷ 19	28,497 ÷ 69	27,898 ÷ 37	80,997 ÷ 87
(18)	2795 ÷ 38	3659 ÷ 57	47,348 ÷ 76	55,697 ÷ 67	44,791 ÷ 47

NOTE TO TEACHER. In some methods of long division, the trial quotient figure has to be corrected from two to five times before the right quotient figure is found. These corrections can be avoided if the following rules are used for all 2-figure divisors from 19 to 99 and for 3-figure divisors from 191 to 999:

For 2-figure divisors ending in 1, 2, 3, 4, or 5 (and for 3-figure divisors having a *second figure* of 1, 2, 3, 4, or 5), divide the partial dividend by the first figure of the divisor to find each quotient figure.

For 2-figure divisors ending in 6, 7, 8, or 9 (and for 3-figure divisors having a *second figure* of 6, 7, 8, or 9), divide the partial dividend by *one more* than the first figure of the divisor to find each quotient figure.

These rules give the correct quotient figure *on the first trial* in over 75% of all cases. In practically all other cases the trial quotient figure has to be corrected *only once* to obtain the right quotient figure.

REMEDIAL SHEET 10

ADDING FRACTIONS

Copy the exercises on a separate sheet of paper. Then add and check:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
(1)	$5\frac{1}{2}$	$8\frac{1}{5}$	$7\frac{1}{5}$	$7\frac{1}{2}$	$2\frac{3}{4}$	$5\frac{4}{5}$	$4\frac{2}{3}$	$2\frac{1}{3}$	$9\frac{1}{8}$	$7\frac{15}{16}$
	$4\frac{1}{2}$	$5\frac{2}{5}$	$8\frac{2}{5}$	3	$9\frac{1}{4}$	$5\frac{2}{5}$	$2\frac{2}{3}$	$8\frac{1}{3}$	$9\frac{7}{8}$	$7\frac{1}{16}$
(2)	$7\frac{3}{8}$	$8\frac{1}{16}$	$1\frac{3}{4}$	$8\frac{5}{8}$	$2\frac{5}{6}$	$9\frac{3}{10}$	$8\frac{7}{8}$	$5\frac{3}{8}$	$1\frac{1}{4}$	$9\frac{7}{16}$
	$5\frac{1}{8}$	$4\frac{3}{16}$	$2\frac{3}{4}$	$3\frac{7}{8}$	$3\frac{5}{6}$	$4\frac{3}{10}$	$7\frac{7}{8}$	$2\frac{3}{8}$	$6\frac{1}{4}$	$4\frac{15}{16}$
(3)	$3\frac{1}{2}$	$8\frac{9}{10}$	$7\frac{5}{6}$	$3\frac{5}{8}$	$6\frac{1}{2}$	$6\frac{4}{5}$	$9\frac{1}{2}$	$4\frac{1}{2}$	$2\frac{7}{8}$	$4\frac{3}{4}$
	$9\frac{1}{4}$	$2\frac{3}{5}$	$5\frac{5}{12}$	$5\frac{5}{16}$	$2\frac{1}{6}$	$5\frac{7}{10}$	$8\frac{7}{16}$	$5\frac{1}{10}$	$6\frac{1}{2}$	$5\frac{9}{16}$
(4)	$8\frac{1}{2}$	$7\frac{1}{3}$	$9\frac{1}{2}$	$4\frac{1}{5}$	$3\frac{2}{3}$	$2\frac{2}{3}$	$2\frac{1}{4}$	$8\frac{1}{2}$	$4\frac{1}{3}$	$2\frac{4}{5}$
	$4\frac{2}{3}$	$1\frac{1}{4}$	$2\frac{4}{5}$	$8\frac{1}{2}$	$6\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{3}{5}$	$7\frac{1}{3}$	$5\frac{3}{4}$	$6\frac{3}{4}$
(5)	$2\frac{1}{3}$	$6\frac{5}{8}$	$3\frac{4}{5}$	$6\frac{2}{3}$	$2\frac{1}{5}$	$4\frac{1}{2}$	$7\frac{3}{5}$	$1\frac{1}{3}$	$4\frac{1}{5}$	$6\frac{1}{4}$
	$2\frac{1}{2}$	$1\frac{1}{3}$	$3\frac{1}{2}$	$4\frac{1}{8}$	$8\frac{2}{3}$	$1\frac{2}{3}$	$3\frac{3}{4}$	$8\frac{1}{8}$	$5\frac{3}{8}$	$3\frac{1}{3}$
(6)	$6\frac{1}{8}$	$8\frac{7}{12}$	$3\frac{1}{2}$	$1\frac{2}{5}$	$8\frac{5}{16}$	$8\frac{3}{16}$	$9\frac{1}{6}$	$7\frac{1}{2}$	$3\frac{3}{4}$	$3\frac{2}{3}$
	$3\frac{1}{4}$	$5\frac{2}{3}$	$8\frac{5}{8}$	$9\frac{1}{2}$	$3\frac{7}{8}$	$5\frac{1}{8}$	$3\frac{2}{3}$	$8\frac{3}{10}$	$6\frac{5}{8}$	$6\frac{5}{6}$
	$3\frac{1}{2}$	$6\frac{1}{4}$	$1\frac{3}{16}$	$4\frac{7}{10}$	$7\frac{3}{32}$	$9\frac{3}{4}$	$8\frac{1}{2}$	$2\frac{4}{5}$	$8\frac{1}{2}$	$9\frac{1}{12}$
(7)	$7\frac{3}{4}$	$1\frac{1}{6}$	$2\frac{1}{3}$	$9\frac{1}{2}$	$2\frac{1}{5}$	$5\frac{1}{2}$	$7\frac{7}{10}$	$5\frac{1}{3}$	$4\frac{7}{8}$	$7\frac{1}{5}$
	$4\frac{1}{2}$	$2\frac{3}{5}$	$9\frac{3}{8}$	$8\frac{1}{6}$	$6\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{3}{5}$	$7\frac{3}{4}$	$9\frac{2}{3}$	$5\frac{3}{4}$
	$9\frac{1}{8}$	$1\frac{2}{3}$	$8\frac{5}{6}$	$7\frac{3}{8}$	$8\frac{3}{4}$	$9\frac{2}{3}$	$9\frac{1}{4}$	$8\frac{5}{6}$	$2\frac{1}{2}$	$3\frac{1}{2}$
	$5\frac{2}{3}$	$8\frac{1}{2}$	$6\frac{5}{12}$	$5\frac{3}{4}$	$4\frac{3}{10}$	$7\frac{5}{8}$	$4\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{10}$

REMEDIAL SHEET 11

SUBTRACTING FRACTIONS

Copy and subtract. Check the work:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
(1)	$6\frac{5}{8}$ $2\frac{1}{8}$	$5\frac{2}{3}$ $1\frac{1}{3}$	$8\frac{5}{6}$ $3\frac{1}{6}$	$9\frac{4}{5}$ $6\frac{2}{5}$	$8\frac{7}{10}$ $2\frac{3}{10}$	$15\frac{5}{12}$ $4\frac{1}{12}$	$13\frac{5}{16}$ $7\frac{3}{16}$	$11\frac{3}{4}$ $9\frac{3}{4}$	$18\frac{7}{8}$ $9\frac{1}{8}$
(2)	$4\frac{7}{8}$ $2\frac{1}{2}$	$9\frac{1}{2}$ $3\frac{1}{4}$	$7\frac{3}{8}$ $1\frac{1}{16}$	$8\frac{11}{12}$ $5\frac{2}{3}$	$6\frac{3}{4}$ $4\frac{5}{12}$	$10\frac{4}{5}$ $3\frac{3}{10}$	$14\frac{3}{4}$ $6\frac{5}{16}$	$10\frac{1}{2}$ $7\frac{1}{10}$	$11\frac{5}{6}$ $2\frac{1}{2}$
(3)	$5\frac{3}{8}$ $4\frac{7}{8}$	$6\frac{1}{4}$ $3\frac{3}{4}$	$9\frac{1}{16}$ $4\frac{5}{16}$	$6\frac{3}{8}$ $5\frac{3}{4}$	$7\frac{1}{4}$ $4\frac{5}{12}$	$12\frac{3}{10}$ $6\frac{1}{2}$	$18\frac{1}{3}$ $4\frac{7}{12}$	$13\frac{2}{3}$ $9\frac{5}{6}$	19 $2\frac{3}{8}$
(4)	$8\frac{5}{8}$ $4\frac{11}{16}$	$5\frac{3}{4}$ $2\frac{7}{8}$	$7\frac{5}{16}$ $6\frac{1}{2}$	$9\frac{1}{6}$ $5\frac{1}{2}$	$4\frac{2}{5}$ $3\frac{7}{10}$	$17\frac{5}{6}$ $9\frac{11}{12}$	$14\frac{1}{3}$ $8\frac{5}{6}$	$11\frac{1}{2}$ $9\frac{7}{10}$	$15\frac{3}{16}$ $6\frac{1}{2}$
(5)	$5\frac{1}{2}$ $4\frac{7}{12}$	$4\frac{1}{8}$ $1\frac{3}{16}$	$8\frac{4}{5}$ $1\frac{9}{10}$	$7\frac{1}{2}$ $5\frac{5}{8}$	$9\frac{3}{8}$ $8\frac{9}{16}$	$10\frac{1}{6}$ $4\frac{1}{3}$	$18\frac{5}{12}$ $7\frac{1}{2}$	$12\frac{3}{10}$ $5\frac{4}{5}$	15 $8\frac{4}{5}$
(6)	$5\frac{3}{5}$ $3\frac{9}{10}$	$8\frac{1}{10}$ $3\frac{1}{2}$	$6\frac{1}{3}$ $4\frac{1}{2}$	$8\frac{5}{12}$ $6\frac{2}{3}$	$9\frac{1}{2}$ $2\frac{7}{8}$	$11\frac{1}{5}$ $7\frac{3}{10}$	$16\frac{1}{12}$ $8\frac{1}{6}$	$13\frac{1}{2}$ $5\frac{3}{5}$	$19\frac{1}{16}$ $6\frac{3}{4}$
(7)	$9\frac{3}{10}$ $7\frac{3}{4}$	$8\frac{1}{3}$ $7\frac{3}{4}$	$7\frac{1}{8}$ $3\frac{9}{16}$	$5\frac{3}{4}$ $2\frac{25}{32}$	$8\frac{2}{3}$ $5\frac{1}{2}$	$13\frac{5}{8}$ $5\frac{3}{4}$	$17\frac{3}{32}$ $2\frac{5}{8}$	$14\frac{1}{2}$ $9\frac{9}{16}$	$12\frac{1}{4}$ $8\frac{3}{8}$
(8)	$7\frac{1}{2}$ $2\frac{2}{3}$	$8\frac{1}{5}$ $1\frac{1}{2}$	$6\frac{3}{4}$ $4\frac{1}{3}$	$9\frac{3}{5}$ $3\frac{1}{3}$	$4\frac{5}{8}$ $2\frac{2}{3}$	$10\frac{1}{2}$ $6\frac{4}{5}$	$12\frac{1}{5}$ $5\frac{1}{6}$	$18\frac{1}{3}$ $9\frac{1}{2}$	$11\frac{1}{4}$ $3\frac{1}{3}$

REMEDIAL SHEET 12

MULTIPLYING AND DIVIDING FRACTIONS

Multiplying Fractions. Copy. Then multiply and check:

	a	*b*	*c*	*d*	*e*	*f*
(1)	$\frac{3}{5} \times \frac{1}{8}$	$\frac{2}{3} \times \frac{1}{5}$	$\frac{3}{4} \times \frac{3}{5}$	$\frac{3}{5} \times \frac{1}{4}$	$\frac{1}{6} \times \frac{1}{5}$	$\frac{5}{8} \times \frac{1}{2}$
(2)	$\frac{3}{8} \times \frac{4}{5}$	$\frac{1}{2} \times \frac{4}{5}$	$\frac{4}{5} \times \frac{15}{16}$	$\frac{2}{3} \times \frac{15}{16}$	$\frac{2}{3} \times \frac{9}{16}$	$\frac{2}{3} \times \frac{9}{8}$
(3)	$\frac{3}{4} \times \frac{4}{5}$	$\frac{2}{3} \times \frac{9}{10}$	$\frac{1}{6} \times \frac{9}{10}$	$\frac{1}{10} \times \frac{15}{16}$	$\frac{5}{12} \times \frac{9}{10}$	$\frac{1}{6} \times \frac{4}{5}$
(4)	$\frac{3}{4} \times 72$	$\frac{5}{6} \times 84$	$\frac{7}{8} \times 56$	$10 \times \frac{5}{16}$	$21 \times \frac{5}{6}$	$25 \times \frac{1}{10}$
(5)	$8 \times \frac{1}{3}$	$5 \times \frac{3}{4}$	$7 \times \frac{5}{6}$	$\frac{1}{5} \times 45$	$\frac{3}{8} \times 64$	$\frac{5}{6} \times 54$
(6)	$1\frac{2}{3} \times 15$	$5\frac{3}{8} \times 72$	$2\frac{1}{6} \times 39$	$9 \times 9\frac{3}{4}$	$7 \times 4\frac{5}{6}$	$3 \times 2\frac{1}{5}$
(7)	$5 \times 2\frac{3}{8}$	$6 \times 8\frac{2}{5}$	$3 \times 2\frac{7}{8}$	$7\frac{2}{3} \times 33$	$1\frac{5}{8} \times 24$	$5\frac{1}{4} \times 30$
(8)	$\frac{5}{6} \times 2\frac{3}{10}$	$\frac{3}{4} \times 2\frac{2}{5}$	$1\frac{1}{3} \times \frac{3}{8}$	$4\frac{1}{6} \times \frac{3}{5}$	$3\frac{3}{5} \times 1\frac{7}{12}$	$1\frac{3}{5} \times 2\frac{3}{16}$
(9)	$\frac{3}{5} \times 1\frac{1}{12}$	$\frac{3}{5} \times 3\frac{3}{4}$	$3\frac{3}{8} \times \frac{2}{3}$	$5\frac{1}{3} \times \frac{3}{10}$	$6\frac{1}{4} \times 1\frac{3}{10}$	$1\frac{1}{5} \times 2\frac{1}{12}$
(10)	$\frac{3}{4} \times 2\frac{5}{6}$	$\frac{1}{6} \times 2\frac{2}{5}$	$1\frac{7}{8} \times \frac{4}{5}$	$2\frac{4}{5} \times \frac{5}{6}$	$1\frac{2}{3} \times 2\frac{1}{10}$	$2\frac{2}{3} \times 1\frac{5}{16}$

Dividing Fractions. Copy. Then divide and check:

(11)	$\frac{5}{16} \div 5$	$\frac{7}{8} \div 3$	$\frac{2}{3} \div 9$	$2\frac{2}{3} \div 6$	$3\frac{3}{8} \div 6$	$2\frac{1}{10} \div 3$
(12)	$6\frac{3}{4} \div 6$	$5\frac{1}{3} \div 4$	$2\frac{4}{5} \div 2$	$\frac{4}{5} \div 8$	$\frac{5}{8} \div 4$	$\frac{3}{16} \div 9$
(13)	$\frac{5}{8} \div \frac{1}{3}$	$4\frac{4}{5} \div \frac{9}{10}$	$\frac{3}{4} \div \frac{7}{12}$	$6 \div \frac{3}{4}$	$6\frac{3}{5} \div \frac{9}{10}$	$14 \div \frac{7}{8}$
(14)	$\frac{7}{12} \div \frac{5}{6}$	$5\frac{1}{3} \div \frac{2}{3}$	$\frac{5}{8} \div \frac{11}{12}$	$8 \div \frac{4}{5}$	$1\frac{1}{2} \div \frac{3}{2}$	$8 \div \frac{2}{3}$
(15)	$\frac{5}{8} \div \frac{3}{4}$	$3\frac{3}{4} \div \frac{5}{8}$	$\frac{3}{8} \div \frac{11}{16}$	$9 \div \frac{3}{10}$	$3\frac{1}{5} \div \frac{1}{10}$	$6 \div \frac{3}{5}$
(16)	$\frac{2}{3} \div 1\frac{2}{5}$	$15 \div 3\frac{1}{3}$	$\frac{7}{8} \div 1\frac{9}{16}$	$3\frac{3}{8} \div 1\frac{4}{5}$	$45 \div 1\frac{7}{8}$	$3\frac{1}{5} \div 2\frac{2}{3}$
(17)	$\frac{5}{6} \div 2\frac{2}{3}$	$28 \div 1\frac{3}{4}$	$\frac{1}{5} \div 3\frac{3}{5}$	$1\frac{3}{5} \div 1\frac{1}{3}$	$18 \div 2\frac{1}{4}$	$6\frac{1}{4} \div 3\frac{1}{3}$
(18)	$\frac{3}{4} \div 1\frac{5}{6}$	$10 \div 1\frac{3}{5}$	$\frac{7}{8} \div 1\frac{7}{8}$	$2\frac{1}{4} \div 1\frac{3}{4}$	$48 \div 1\frac{3}{5}$	$2\frac{4}{5} \div 1\frac{3}{4}$

MULTIPLYING AND DIVIDING DECIMALS

Multiplying Decimals. Copy. Then multiply and check:

	a	*b*	*c*	*d*	*e*
(1)	$6 \times .078$	$9 \times .087$	27×9.308	58×9.2	$26 \times .079$
(2)	$9 \times .096$	$6 \times .009$	62×8.639	67×5.4	$38 \times .368$
(3)	4×3.28	$5 \times .078$	74×7.946	$74 \times .87$	$92 \times .084$
(4)	8×9.27	8×43.8	69×45.32	$85 \times .56$	89×67.1
(5)	7×6.74	3×9.27	86×64.52	$69 \times .35$	43×5.14
(6)	6.1×8.9	4.8×5.8	$3.8 \times .74$	$2.8 \times .315$	$9.8 \times .324$
(7)	$.38 \times 27$	$1.9 \times .18$	5.6×6.5	$6.3 \times .089$	9.9×6.89
(8)	$.56 \times .47$	$.89 \times .39$	$.76 \times 1.8$	$.42 \times 89.2$	$.15 \times 75.2$
(9)	$.94 \times 56$	$.57 \times 9.3$	$.47 \times .52$	$.16 \times .049$	$.27 \times 5.06$
(10)	$4.3 \times .94$	$.08 \times 4.6$	$.03 \times .315$	$.078 \times 92.8$	$.84 \times .061$
(11)	$.06 \times .63$	$.04 \times 3.7$	$.09 \times .027$	$.096 \times 50.3$	$.16 \times .271$
(12)	$.09 \times .47$	$.05 \times .91$	$.06 \times .007$	$.007 \times 4.97$	$.85 \times 4.23$
(13)	$.07 \times .07$	$.07 \times 2.7$	$.05 \times .008$	$.009 \times 2.19$	$.79 \times 7.53$

Dividing Decimals. Copy and divide, annexing 0's to the dividend as needed:

(14)	$4\overline{)9.32}$	$6\overline{)4.596}$	$9\overline{)8.487}$	$6\overline{)56.52}$	$8\overline{)71.632}$
(15)	$5\overline{)7.35}$	$4\overline{)6.252}$	$7\overline{).6482}$	$8\overline{)23.76}$	$6\overline{).969924}$
(16)	$8\overline{)9.52}$	$9\overline{)32.04}$	$3\overline{).7371}$	$5\overline{).6965}$	$7\overline{)8.7969}$
(17)	$4\overline{)201}$	$6\overline{)3.813}$	$5\overline{)32.82}$	$2\overline{).3017}$	$8\overline{)469.3}$
(18)	$8\overline{)628}$	$4\overline{)37.41}$	$2\overline{)1.901}$	$6\overline{)14.91}$	$5\overline{)23.81}$
(19)	$5\overline{)196}$	$2\overline{)5.967}$	$8\overline{)116.9}$	$4\overline{).2757}$	$6\overline{).2937}$
(20)	$2\overline{)937}$	$8\overline{)23.65}$	$6\overline{)442.5}$	$5\overline{).4396}$	$4\overline{)11.35}$

REMEDIAL SHEET 14

DIVIDING DECIMALS

Avoiding Remainders. Copy and divide. Annex enough 0's after the dividend to make the quotient come out exact:

	a	*b*	*c*	*d*	*e*
(1)	116 ÷ 25	749 ÷ 35	4472 ÷ 65	404.6 ÷ 85	201.6 ÷ 45
(2)	738 ÷ 36	301 ÷ 14	4389 ÷ 66	104.4 ÷ 72	130.9 ÷ 35
(3)	209 ÷ 44	432 ÷ 32	3013 ÷ 46	460.1 ÷ 86	39.99 ÷ 86
(4)	615 ÷ 75	585 ÷ 78	4607 ÷ 85	166.1 ÷ 55	20.14 ÷ 95
(5)	144 ÷ 64	342 ÷ 72	2985 ÷ 75	264.6 ÷ 84	218.4 ÷ 48

Rounding Off Quotients. Copy and divide. Carry the quotient to two decimal places; then round off the result correct to the nearest tenth:

(6)	473 ÷ 35	173 ÷ 26	123.2 ÷ 17	77.51 ÷ 83	1852.6 ÷ 298
(7)	821 ÷ 58	294 ÷ 19	409.6 ÷ 21	284.7 ÷ 34	407.69 ÷ 321
(8)	987 ÷ 92	637 ÷ 41	76.32 ÷ 58	84.99 ÷ 97	2941.1 ÷ 472
(9)	259 ÷ 67	428 ÷ 83	24.07 ÷ 49	612.4 ÷ 24	3099.7 ÷ 538
(10)	836 ÷ 39	196 ÷ 45	141.5 ÷ 76	388.7 ÷ 53	277.43 ÷ 146

Rounding Off Quotients. Copy and divide. Carry the quotient to three decimal places; then round off the result correct to the nearest hundredth:

(11)	457 ÷ 81	24.7 ÷ 68	236.4 ÷ 23	146.9 ÷ 19	2348.4 ÷ 418
(12)	331 ÷ 46	68.5 ÷ 93	269.1 ÷ 87	31.82 ÷ 22	7162.4 ÷ 873
(13)	462 ÷ 59	8.82 ÷ 18	85.73 ÷ 42	30.75 ÷ 36	168.98 ÷ 325
(14)	147 ÷ 16	5.37 ÷ 45	291.4 ÷ 89	217.9 ÷ 43	528.77 ÷ 254
(15)	268 ÷ 73	27.4 ÷ 38	38.77 ÷ 64	121.5 ÷ 17	1054.4 ÷ 112

Dividing by a Decimal. Copy and divide. Check by multiplying:

	a	*b*	*c*	*d*
(16)	\$9.36 ÷ \$.24	\$90.24 ÷ \$2.82	\$93.44 ÷ \$5.84	\$95.38 ÷ \$5.02
(17)	\$9.72 ÷ \$.36	\$47.52 ÷ \$0.48	\$80.75 ÷ \$0.19	\$97.23 ÷ \$4.63
(18)	\$9.48 ÷ \$.79	\$45.88 ÷ \$0.62	\$71.53 ÷ \$3.11	\$83.16 ÷ \$3.08
(19)	\$9.79 ÷ \$.89	\$98.93 ÷ \$7.61	\$77.35 ÷ \$4.55	\$89.01 ÷ \$0.69
(20)	\$8.67 ÷ \$.51	\$51.66 ÷ \$1.26	\$99.58 ÷ \$7.66	\$75.71 ÷ \$1.13

DIVIDING BY A DECIMAL

Exact Division. Copy and divide. Check the work by multiplying:

	a	*b*	*c*	*d*
(1)	39.75 ÷ .75	13.744 ÷ 1.6	26.215 ÷ .35	192.786 ÷ 3.81
(2)	9.956 ÷ .19	336.68 ÷ 3.8	140.94 ÷ 2.7	28.7625 ÷ .885
(3)	8.892 ÷ .26	19.006 ÷ .34	35.796 ÷ 3.8	3.04164 ÷ .426
(4)	93.38 ÷ .46	2.8609 ÷ .61	261.12 ÷ .64	18.5592 ÷ 22.8

Avoiding Remainders. Divide. Annex enough 0's after the dividend to make the quotient come out exact. Check:

(5)	.304 ÷ .64	.156 ÷ .016	4.758 ÷ .065	5.312 ÷ 1.28
(6)	.744 ÷ .96	.576 ÷ 2.56	10.92 ÷ .112	3.9168 ÷ 5.12
(7)	50.49 ÷ .55	6.16 ÷ 22.4	73.92 ÷ 19.2	293.44 ÷ 44.8
(8)	4.296 ÷ .48	.2096 ÷ .032	2.436 ÷ 3.36	25.152 ÷ .384

Approximate Quotients. Divide. Carry the quotient to three decimal places; then round off the result correct to the nearest hundredth:

(9)	96.47 ÷ 5.3	4.449 ÷ 5.9	4.816 ÷ 6.5	7.484 ÷ 5.9
(10)	72.53 ÷ 8.7	8.704 ÷ 6.5	65.22 ÷ 3.3	12.85 ÷ 4.9
(11)	64.72 ÷ 8.6	9.027 ÷ 5.8	5.911 ÷ .18	19.27 ÷ 2.8
(12)	86.38 ÷ 1.8	17.18 ÷ 8.4	13.88 ÷ 3.9	1.0926 ÷ .97
(13)	5.682 ÷ .43	26.75 ÷ 4.5	3.7188 ÷ .41	5.8601 ÷ .62
(14)	4.381 ÷ .51	35.59 ÷ 7.1	3.8144 ÷ .26	24.864 ÷ .88

Zero Difficulties. Divide, annexing 0's to the dividend as needed. Check:

(15)	.2294 ÷ 6.2	.6882 ÷ 7.4	84.56 ÷ .604	1.9725 ÷ .025
(16)	.1885 ÷ 6.5	.6318 ÷ 8.1	834.1 ÷ .439	1254.4 ÷ 1.96
(17)	231.4 ÷ .26	.5141 ÷ 9.7	912.6 ÷ 7.02	1.3572 ÷ 23.4
(18)	151.3 ÷ .17	170.2 ÷ .37	1.656 ÷ .036	132.25 ÷ .023
(19)	.2565 ÷ 4.5	817.8 ÷ .94	292.1 ÷ .046	4621.5 ÷ 5.85

FRACTIONS, DECIMALS, AND PER CENTS

Changing Fractions to Decimals. Copy; then change to exact decimals:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
(1)	$\frac{29}{50}$	$\frac{9}{16}$	$\frac{7}{25}$	$\frac{3}{20}$	$\frac{18}{25}$	$\frac{3}{16}$	$\frac{21}{25}$	$\frac{17}{50}$	$\frac{1}{16}$	$\frac{14}{25}$	$\frac{11}{16}$	$\frac{13}{25}$	$\frac{13}{16}$

Change to decimals, correct to the nearest hundredth:

(2)	$\frac{5}{9}$	$\frac{3}{7}$	$\frac{29}{36}$	$\frac{17}{32}$	$\frac{17}{31}$	$\frac{11}{14}$	$\frac{1}{12}$	$\frac{51}{67}$	$\frac{16}{35}$	$\frac{37}{53}$	$\frac{37}{42}$	$\frac{18}{41}$	$\frac{17}{36}$
(3)	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{19}{24}$	$\frac{25}{28}$	$\frac{11}{17}$	$\frac{11}{45}$	$\frac{24}{39}$	$\frac{31}{36}$	$\frac{41}{42}$	$\frac{19}{61}$	$\frac{11}{32}$	$\frac{24}{59}$	$\frac{29}{63}$
(4)	$\frac{6}{7}$	$\frac{7}{9}$	$\frac{13}{15}$	$\frac{14}{19}$	$\frac{11}{36}$	$\frac{23}{29}$	$\frac{14}{53}$	$\frac{17}{23}$	$\frac{23}{41}$	$\frac{13}{28}$	$\frac{15}{71}$	$\frac{28}{43}$	$\frac{22}{37}$

Changing Fractions to Per Cents. Copy; then change to per cents:

(5)	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{4}{5}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{1}{5}$	$\frac{5}{8}$	$\frac{1}{2}$

Changing Per Cents to Decimals. Copy; then change to decimals:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
(6)	66%	27%	9%	$12\frac{1}{2}\%$	102%	9.7%	43.1%	4.67%
(7)	89%	99%	2%	$15\frac{3}{4}\%$	427%	6.4%	11.4%	8.33%
(8)	18%	37%	7%	$22\frac{1}{2}\%$	500%	0.5%	13.7%	0.17%
(9)	71%	50%	5%	$41\frac{3}{4}\%$	980%	1.1%	19.9%	0.09%

Changing Decimals to Per Cents. Copy; then change to per cents:

(10)	.47	.10	.03	$.03\frac{1}{4}$	1.67	.048	.107	.0462
(11)	.34	.08	.17	$.04\frac{1}{2}$	3.00	.071	.004	.1525
(12)	.40	.04	.66	$.11\frac{1}{2}$	6.00	.003	.601	.0003
(13)	.83	.77	.11	$.17\frac{3}{4}$	7.12	.152	.732	.0025

REMEDIAL SHEET 17

PERCENTAGE

Finding Per Cents. Copy; then find the following per cents:

	a	*b*	*c*	*d*
(1)	52% of $85	65% of $260	4% of $6.50	3% of $1270
(2)	38% of $72	82% of $455	7% of $8.00	4% of $4880
(3)	17% of $55	12% of $144	5% of $4.20	5% of $3375
(4)	62% of $90	19% of $170	6% of $8.50	6% of $2500
(5)	95% of $88	26% of $395	8% of $4.00	7% of $2250
(6)	81% of $65	58% of $150	2% of $7.50	4% of $1200

Computing to the Nearest Cent. Find results correct to the nearest cent:

(7)	6% of $9.75	3% of $12.60	4% of $50.75	10% of $200.75
(8)	2% of $6.89	5% of $15.75	8% of $77.62	21% of $650.80
(9)	8% of $7.25	6% of $18.69	3% of $50.25	19% of $722.50
(10)	4% of $8.68	2% of $42.55	2% of $16.65	22% of $150.25
(11)	7% of $5.35	3% of $85.75	4% of $75.25	28% of $320.20
(12)	3% of $8.75	4% of $57.35	5% of $50.50	15% of $450.75
(13)	5% of $9.78	5% of $84.43	7% of $60.15	18% of $550.25

Short Cuts in Percentage. Find the following correct to the nearest cent:

(14)	50% of $89	75% of $3.36	$33\frac{1}{3}$% of $19.64	$37\frac{1}{2}$% of $13.70
(15)	25% of $28	25% of $9.30	$12\frac{1}{2}$% of $15.15	$66\frac{2}{3}$% of $17.91
(16)	80% of $75	10% of $4.95	$62\frac{1}{2}$% of $37.13	$16\frac{2}{3}$% of $17.70
(17)	75% of $69	75% of $8.24	$37\frac{1}{2}$% of $14.40	$33\frac{1}{3}$% of $13.57
(18)	60% of $90	50% of $4.35	$87\frac{1}{2}$% of $42.45	$83\frac{1}{3}$% of $30.96

Changing Per Cents to Fractions. Change to common fractions:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
(19)	25%	$37\frac{1}{2}$%	$16\frac{2}{3}$%	50%	40%	$62\frac{1}{2}$%	10%	$87\frac{1}{2}$%	80%
(20)	60%	$83\frac{1}{3}$%	$12\frac{1}{2}$%	75%	70%	$66\frac{2}{3}$%	50%	$33\frac{1}{3}$%	25%

REMEDIAL SHEET 18

PERCENTAGE

Per Cents Larger Than 100%. Copy; then find the following:

	a	*b*	*c*	*d*
(1)	150% of \$260	375% of \$630	215% of \$390	150% of \$960
(2)	500% of \$860	133% of \$674	145% of \$352	110% of \$225
(3)	196% of \$750	175% of \$272	200% of \$328	180% of \$250
(4)	212% of \$645	230% of \$450	325% of \$200	400% of \$885

Per Cents Ending in Fractions. Find the following correct to the nearest cent:

(5)	$3\frac{1}{4}\%$ of \$15.75	$4\frac{1}{3}\%$ of \$12.50	$2\frac{3}{4}\%$ of \$42.50	$1\frac{3}{4}\%$ of \$84.40
(6)	$4\frac{1}{4}\%$ of \$86.50	$2\frac{3}{4}\%$ of \$62.50	$3\frac{1}{4}\%$ of \$12.60	$4\frac{2}{3}\%$ of \$85.75
(7)	$2\frac{1}{2}\%$ of \$15.85	$6\frac{1}{3}\%$ of \$50.75	$6\frac{1}{2}\%$ of \$57.25	$6\frac{1}{2}\%$ of \$48.75
(8)	$5\frac{2}{3}\%$ of \$76.50	$3\frac{3}{4}\%$ of \$38.50	$3\frac{1}{2}\%$ of \$18.75	$4\frac{3}{4}\%$ of \$28.50

Harder Per Cents. Find the following correct to the nearest cent:

	a	*b*	*c*
(9)	3.4% of \$712	1.3% of \$1500	5.27% of \$140.40
(10)	0.5% of \$624	4.1% of \$3580	1.15% of \$198.60
(11)	4.4% of \$256	2.5% of \$4260	2.36% of \$195.70
(12)	2.8% of \$524	1.4% of \$1650	1.34% of \$268.50

Per Cents Smaller Than 1%. Find the following correct to the nearest cent:

(13)	$\frac{1}{3}$ of 1% of \$360	$\frac{1}{2}$ of 1% of \$455	0.5% of \$3750
(14)	$\frac{1}{10}$ of 1% of \$928	$\frac{3}{5}$ of 1% of \$625	0.9% of \$7850
(15)	$\frac{1}{5}$ of 1% of \$475	$\frac{1}{4}$ of 1% of \$900	0.3% of \$6800
(16)	$\frac{1}{4}$ of 1% of \$200	$\frac{2}{3}$ of 1% of \$850	0.4% of \$8175

PERCENTAGE

Comparing Numbers. Find what per cent the first number is of the second:

	a	*b*	*c*	*d*	*e*	*f*
(1)	9, 25	46, 92	84, 48	39, 39	25, 200	125, 500
(2)	7, 20	23, 25	65, 52	36, 75	35, 280	185, 222
(3)	3, 50	96, 64	65, 78	11, 66	87, 116	175, 100
(4)	6, 80	35, 75	64, 96	14, 16	93, 248	192, 384

Find, to the nearest tenth of 1%, the per cent that the first number is of the second:

(5)	7, 27	14, 27	19, 26	20, 37	99, 140	137, 264
(6)	9, 22	42, 89	15, 24	27, 33	69, 420	160, 280
(7)	7, 74	34, 75	18, 13	11, 18	77, 150	128, 340
(8)	6, 19	17, 62	47, 83	35, 24	48, 225	191, 500

Finding the Base. Find the number that should be put on each line:

	a	*b*	*c*
(9)	6 is 3% of _____	58 is 2% of _____	135 is 18% of _____
(10)	7 is 4% of _____	90 is 6% of _____	102 is 12% of _____
(11)	5 is 1% of _____	85 is 5% of _____	473 is 11% of _____
(12)	8 is 4% of _____	27 is 9% of _____	128 is 16% of _____

Per Cents of Increase or Decrease. These pairs of numbers represent for two successive years the populations of a town or the profits of a store. Find the per cent of increase or decrease that the second number shows over the first number:

	a	*b*	*c*	*d*
(13)	4320, 3240	2400, 2274	3600, 3756	2750, 2970
(14)	2400, 2796	3870, 6966	3920, 2940	1500, 3150
(15)	1550, 1240	1530, 2448	1440, 1656	3950, 4503
(16)	3500, 7000	4824, 3618	2500, 2200	6700, 6030
(17)	4840, 4598	3848, 3367	6350, 6223	1888, 2360
(18)	6228, 9342	5500, 6050	2875, 2990	4500, 4365

TABLES OF WEIGHTS AND MEASURES

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$16\frac{1}{2}$ ft. or $5\frac{1}{2}$ yd. = 1 rod (rd.)
5280 ft. or 320 rd. = 1 mile (mi.)
6080 ft. or 1.15 mi. = 1 nautical mile
6 feet = 1 fathom

Square Measure

144 square (sq.) inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
43,560 square feet or 160 square rods = 1 acre (A.)
640 acres = 1 square mile

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)
2240 pounds = 1 long ton

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference
90 degrees = 1 right angle

Useful Equivalents

1 bushel = about 2150 cu. in. or about $1\frac{1}{4}$ cu. ft.
1 cubic foot = about 0.8 bu.
1 bu. wheat = 60 lb.
1 bu. potatoes = 60 lb.
1 bu. oats = 32 lb.
1 bu. corn = 56 lb.
1 bu. apples = 50 lb.
1 barrel flour = 196 lb.

1 U. S. gallon = 231 cu. in.
1 imperial gallon = 277.4 cu. in.
1 cubic foot = about $7\frac{1}{2}$ gal.
1 ton hard coal = about 35 cu. ft.
1 ton soft coal = about 42 cu. ft.
1 cord of wood = 128 cu. ft.
1 cu. ft. water = about $62\frac{1}{2}$ lb.
1 cu. ft. ice = about $57\frac{1}{2}$ lb.
π = 3.1416, or $3\frac{1}{7}$

Metric Equivalents

1 meter = 39.37 in. or 1.1 yd.
1 kilometer = 0.621 mi.
1 liter = 1.06 liquid qt. or 0.91 dry qt.
1 kilogram = 2.2 lb.
1 metric ton = 2205 lb.

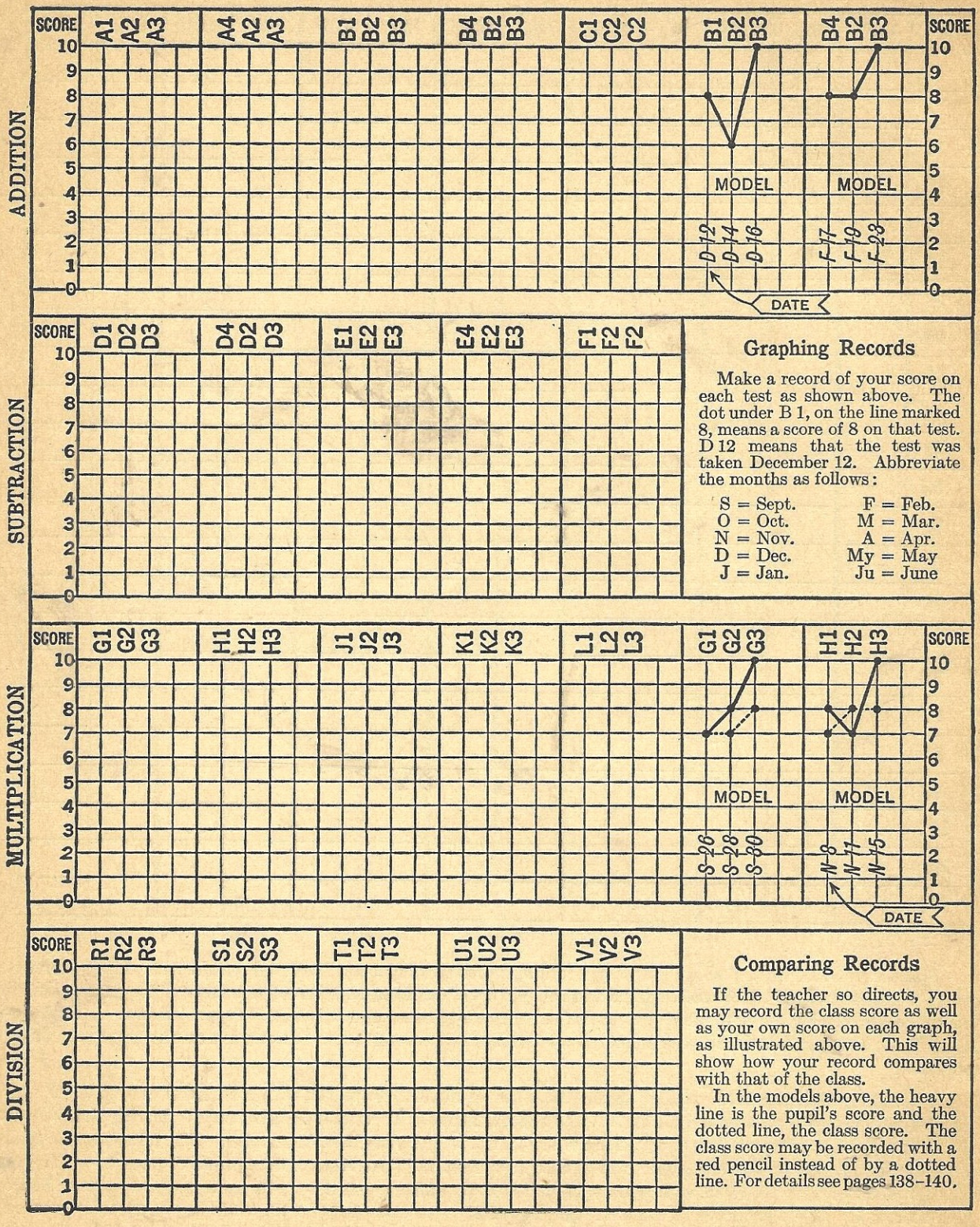
1 yard = 0.914 meter
1 mile = 1.609 kilometer
1 dry quart = 1.10 liter
1 liquid quart = 0.95 liter
1 pound = 0.454 kilogram

PAGE	SCORE	PAGE	SCORE	PAGE	SCORE	PAGE	SCORE	PAGE	SCORE	PAGE	SCORE	PAGE	SCORE
1		109											
2		116											
3		121											
4		131											
6		136											
15		137											
21													
29													
36													
41													
47													
50													
57													
62													
70													
71													
79													
87													
92													
93													
101													
102													
108													

Space is provided on the above chart for recording scores on Diagnostic Tests and certain other sets of exercises where scores are required. In recording the score on any set of exercises, also write the number of the page on which the set occurs, placing the page number to the left of the score in the column headed "Page." Scores on Improvement Tests, however, should not to be entered above since such scores are to be recorded on the graphic chart on page 166.

The scoring of other sets of exercises, besides those just mentioned, is *optional* with the teacher or student; if such scores are found, they may be written in the blank spaces above. To score such sets, use the Scoring Table if the set contains from 2 to 20 exercises, or compute the score if the set has more than 20 exercises. The following examples illustrate the method of computing scores as applied to the set of 35 exercises at the bottom of page 7: (1) If you have 29 exercises right out of 35, your score is $\frac{29}{35}$ of 10, or $8\frac{2}{7}$, which is called 8 to the nearest whole number; hence your score is 8. (2) If you have 31 exercises right out of 35, your score is $\frac{31}{35}$ of 10, which equals $8\frac{6}{7}$; this is called 9 to the nearest whole number. (3) If you have 34 exercises right out of 35, your score is $\frac{34}{35}$ of 10, or $9\frac{5}{7}$, which is called 10 to the nearest whole number.

GRAPHS OF SCORES ON IMPROVEMENT TESTS



Graphing Records

Make a record of your score on each test as shown above. The dot under B 1, on the line marked 8, means a score of 8 on that test. D 12 means that the test was taken December 12. Abbreviate the months as follows:

S = Sept.	F = Feb.
O = Oct.	M = Mar.
N = Nov.	A = Apr.
D = Dec.	My = May
J = Jan.	Ju = June

Comparing Records

If the teacher so directs, you may record the class score as well as your own score on each graph, as illustrated above. This will show how your record compares with that of the class.

In the models above, the heavy line is the pupil's score and the dotted line, the class score. The class score may be recorded with a red pencil instead of by a dotted line. For details see pages 138–140.

NOTE TO TEACHER. Only scores on Improvement Tests should be recorded above in graphic form.

HOW TO FIND YOUR SCORE

In this book, Improvement Tests, Diagnostic Tests, and certain other sets of exercises should be scored. Such sets are easily recognized since spaces for recording the scores are provided under these sets. Your score on any set will be 10 if you get all the exercises right, but it will be less than 10 if you have some of the exercises wrong.

To find your score, proceed as follows: Suppose you have worked the Improvement Test on page 11 and that you got 14 exercises right. Below that test, on the left, there is a short line like this: _____ RIGHT OUT OF 15. This means that the set contains 15 exercises in all. Since you got 14 exercises right, put 14 in the space. On the right side of the page below the test, there is another short line like this: SCORE _____ LINE N. This means that you will find your score in Line N of the Scoring Table below. Line N is the horizontal line shown by Arrow AA. Since you got 14 exercises right, look along Line N until you reach the column with 14 at the top (shown by Arrow BB) and you will find the number 9 (shown by Arrow CC). This means that your score is 9. Then write your score, 9, in the space on page 11.

If you wish to score sets of exercises other than those mentioned above, the Scoring Table may be used for this purpose provided the set does not contain over 20 exercises. For example, to find your score on a set of 14 exercises in which you got 11 exercises right, look for 14 in the column headed "No. of Examples in Set"; then move to the left on the horizontal line marked 14, until you come to the column headed 11 where you will find the number 8 which is your score on that set. For directions for scoring sets containing more than 20 exercises, see the bottom of page 165.

After you finish a set of examples, the teacher will read the answers. In counting the number of exercises that you got right, count only those that are entirely right and entirely finished.

SCORING TABLE

LINE	NUMBER OF EXAMPLES RIGHT																					NO. OF EXAMPLES IN SET	LINE
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
A	0	5	10																			2	A
B	0	3	7	10													BB					3	B
C	0	3	5	8	10																	4	C
D	0	2	4	6	8	10																5	D
E	0	2	3	5	7	8	10															6	E
F	0	1	3	4	6	7	9	10														7	F
G	0	1	3	4	5	6	8	9	10													8	G
H	0	1	2	3	4	6	7	8	9	10												9	H
J	0	1	2	3	4	5	6	7	8	9	10											10	J
K	0	1	2	3	4	5	5	6	7	8	9	10										11	K
L	0	1	2	2	3	4	5	6	7	8	8	9	10									12	L
M	0	1	1	2	3	4	4	5	6	6	7	8	9	9	10			CC				14	M
AA → N	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10						15	N
P	0	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10					16	P
Q	0	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10				17	Q
R	0	1	1	2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	10			18	R
S	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10		19	S
T	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	20	T

Index The Numbers in heavier type Refer to Pages of Remedial

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